C wing geometric redesign based on aircraft flying qualities

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ABSTRACT

This work formulates the application of a multi-objective design methodology for the conceptual design of a C-wing tip for a flying wing powered by an electric power plant. The design objective is to propose C-wing geometric configurations regarding the longitudinal flying qualities. The optimization objectives consider the short-period mode frequency and the time it would take to duplicate the amplitude of the oscillations after being disturbed. A Multi-objective evolutionary algorithm (MOEA) that uses the so-called meta-heuristic Differential Evolution solves the multi-objective optimization problem. The selection of the solutions representing the best compromise between the optimization goals is based on Pareto dominance.

1 INTRODUCTION

Advances in various technological areas have fostered industry and research centers to search for new solutions or improvements to existing ones constantly. In most cases, the goal of innovation can be translated into an optimization problem. In particular, in Aeronautical engineering, aircraft design is, by nature, multi-objective and multidisciplinary, with several conflicting design objectives. Motivated by economic and industrial requirements, aircraft design has experienced innovative changes and, during the last decades, has been considerably improved by using computer simulations [17, 18, 2]. For example, computational fluid dynamics (CFD) for aerodynamic analysis [13, 14, 20, 19], and finite element method (FEM) for structural analysis [15] and [11, 29]. The continuous demand for better designs, subject to economic and environmental constraints, and the continuous development in computer simulation techniques have led to highly computerized aircraft design processes. Unmanned Aerial Vehicles (UAVs) of any size have proven effective in various applications, raising great interest in developing new configurations. Configurations are limited only by the design team's imagination during the conceptual design phase. This high degree of freedom makes the decision-making process complicated. Thus, optimization techniques are of high value in these design stages. In particular, multi-objective optimization techniques are preferred as they provide a set of solutions instead of a single solution. The set of solutions represents the best compromise between the different design objectives such that the decision-making can be quantified.

The flying qualities of human-crewed aircraft have been studied for a long time based on the pilot's evaluation [31]. Their study has achieved a high degree of maturity. It is known with precision the qualities an aircraft must satisfy based on its mission profile, and there is an aircraft classification regarding flying qualities [5]. UAVs' flight qualities must be radically different from the definition of the flying qualities for full-scale aircraft since UAVs are pilotless. Some efforts were made in the seventies to determine flying qualities for remotely piloted aircraft. These efforts tried to define the flying qualities of UAVs, simply scaling the flying qualities of a complete-scale aircraft [24]. Recent efforts to define UAV flying qualities have been reported in [7, 4, 27, 16].

This paper addresses the conceptual redesign of a C-wing tip geometry for a small flying wing powered by an electric power plant using a multi-objective optimization methodology. The design explores the relationship between the C-wing geometric characteristics and the resulting aircraft dynamic characteristics expressed in the flying qualities. The design objective is stated as an optimization problem in terms of the short-period mode frequency and the time it would take to duplicate the amplitude of the oscillations after disturbances. A Multi-objective evolutionary algorithm based on the meta-heuristic differential evolution solves the optimization problem. The selection of the solutions that represent the best compromise between the optimization goals is based on Pareto dominance.

This paper has the following structure. Section 2 describes the multi-objective optimization procedure and presents the study's case. Section 3 introduces the objective functions, establishes the optimization problem, and describes the solution algorithm. Section 4 reports the results obtained from the optimization algorithm. Finally, Section 5 reports concluding remarks.

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2 Multi-objective Optimization

A multi-objective optimization problem can be defined mathematically as

$$\text{minimize } \vec{f}(\vec{x}) := [f_1(\vec{x}), f_2(\vec{x}), \dots, f_k(\vec{x})] \quad (1)$$

constrained to:

$$g_i(\vec{x}) \le 0 \quad i = 1, 2, \dots, m \quad (2)$$

$$h_i(\vec{x}) = 0 \quad i = 1, 2, \dots, p \quad (3)$$

where $\vec{x} = [x_1, x_2, \dots, x_n]^T$ is the decision variables vector, which are bounded by lower limits (x_i^l) and upper limits (x_i^u) defining the search space $\mathcal{S}$. $f_i : \mathbb{R}^n \to \mathbb{R}$, $i = 1, ..., k$ are the objective functions and $g_i, h_j : \mathbb{R}^n \to \mathbb{R}$, $i = 1, ..., m$, $j = 1, ..., p$ are the constraint functions. A minimization problem is assumed to be addressed without loss of generality.

A multi objective problem aims to determine from a set of possible solutions $\mathcal{F}$, satisfying the constraints, those corresponding to optimal values for all k objective functions simultaneously [9]. Any decision variable vector $\vec{x}$ that satisfies the constraints is considered a feasible solution. Pareto dominance is an important ingredient for the notion of optimality in multi- objective optimization problems. To define the Pareto dominance the following definitions are introduced.

Definition 1 *A decision variable vector $\vec{x}$ dominates a second one $\vec{y}$, denoted $\vec{x} \preceq \vec{y}$, if and only if $\vec{x}$ is partially less than $\vec{y}$, in the following sense*

$$\forall\, i \in \{1, \dots, k\},\ f_i(\vec{x}) \le f_i(\vec{y}) \wedge \exists\, i \in \{1, \dots, k\} \ : \ f_i(\vec{x}) < f_i(\vec{y}).$$

Definition 2 *A decision variable vector $\vec{x} \in \mathcal{S} \subset \mathbb{R}^n$ is* **not***-dominated with respect to $\mathcal{S}$, if there is no other vector $\vec{x}' \in \mathcal{S}$ such that $\vec{f}(\vec{x}') \preceq \vec{f}(\vec{x})$.*

To ensure that a solution **dominates** another, it is necessary that the dominating solution is strictly better in at least one design objective and not worse in any of the other design objectives. Thus, for a multi-objective optimization problem, when a comparison between two different solutions $\vec{x}$ and $\vec{y}$ is made, the following possibilities arise [12], [3].

- $\vec{x} \preceq \vec{y}$, $\vec{x}$ dominates $\vec{y}$. $\vec{x} \succeq \vec{y}$, $\vec{x}$ is dominated by $\vec{y}$.
- $\vec{x} \npreceq \vec{y} \wedge \vec{y} \npreceq \vec{x}$, $\vec{x}$ and $\vec{y}$ are not comparable, it is not possible to establish which solution dominates.

Definition 3 *A decision variable vector $\vec{x}^* \in \mathcal{F} \subseteq \mathcal{S}$ is a* **Pareto Optimal** *if it is non-dominated with respect to $\mathcal{F}$.*

A Pareto optimum is a solution that cannot be improved concerning an objective function without worsening other. Hence, a Pareto optimum does not provide a unique solution for the space of the decision variables but provides a set of solutions that form the so-called *Pareto Optimal Set* $\mathcal{P}^*$.

Definition 4 *The* **Pareto Optimal Set** $\mathcal{P}^*$ *is determined as*

$$\mathcal{P}^* = \{\vec{x} \in \mathcal{F} \,|\, \vec{x} \text{ is a Pareto Optimal}\}$$

The solutions contained in $\mathcal{P}^*$ are non-dominated *between them.*

All non-dominated solutions plotted in the objective function space trace the Pareto Optimal called *Pareto front* $\mathcal{PF}^*$.

Definition 5 *The* **Pareto front** $\mathcal{PF}^*$ *is described as*

$$\mathcal{PF}^* = \{\vec{f}(\vec{x}) \in \mathbb{R}^k \,|\, \vec{x} \in \mathcal{P}^*\}$$

The goal in solving multi-objective problems is to determine $\mathcal{P}^*$. Hence, in solving a multi-objective problem, there is no interest in finding a unique solution but the set of solutions representing the best possible compromise between the problem's objectives. Because, in general, it is impossible to find an analytical expression that defines the Pareto front of a multi-objective problem, the commonly used approach is to compute a sufficient number of points in the feasible region, which are then filtered based on the Pareto dominance, to thereby obtaining an approximation of the Pareto front shape.

2.1 The case of study

The aerial platform of this work is a flying wing equipped with a C-wing tip, as shown in Figure 1. This aircraft configuration is derived from [10], where a flying wing with a conventional winglet was designed for aerodynamic parameter identification purposes. The resulting flying wing was unstable because reflex aerodynamic profiles were not used, hindering its radio control human-crewed operation. Here, it is proposed to use a C-wing tip to solve this problem. The C-wing dimensioning problem to reduce aerodynamic drag under different optimization criteria has been addressed in [23, 26, 22]. In [23], the C-wing dimensions were optimized to reduce the aerodynamic drag constrained to generate a statically stable pitching moment. The C-wing tip may help to change the slope of the pitching moment coefficient modifying the aircraft stability properties [23]; consequently, the difficulties in flying the aircraft may be reduced. However, the next question arises: how much this slope needs to be modified? This work proposes a partial answer to that question by redesigning the C-wing geometric dimensions in terms of the longitudinal aircraft flying qualities; this is the main innovation of this work.

3 Optimization problem

The role of the objective functions is to link the C-wing tip geometry with the dynamic aircraft characteristics. Since the aircraft aerodynamic coefficients depend on the aircraft geometry and the dynamic aircraft characteristics depend on the aerodynamic coefficients, the aircraft longitudinal dynamics is an excellent candidate to link all design variables. The

CS chord	0.3m	Wing root chord	0.3m
Wingtip chord	0.15m	Winglet tip chord	0.1m
C tip chord	0.0579m	CS ap	NACA0018
Wing section ap	Eppler 66	C wing ap	NACA 0009
Winglet ap	NACA 009	i_{CS}	0^o
i_{WS}	0^o	i_{wn}	0^o
i_C	-10^o	b_{CS}	0.25m
b_{WS}	0.8m	b_C	0.2215m
Λ_{WS}	52.7515^o	Λ_{wn}	32.5785^o
Λ_C	-29.0231^o	h_{wn}	0.15m

Table 1: Geometric characteristics of the flying wing base design. i incidence, Λ sweep, b span, ap aerodynamic profile. CS central section, WS wing section, wn winglet, C C-wing, h height.

Figure 1: Geometry of the UAV with a C wing tip.

aircraft longitudinal dynamics is described by the following equations [25]

$$\begin{aligned} m\dot{V} &= T\cos(\alpha) - \frac{1}{2}\rho V^2 S C_D(\alpha) - mg\sin(\gamma), \\ mV\dot{\gamma} &= -T\sin(\alpha) + \frac{1}{2}\rho V^2 S C_L(\alpha, q, \delta_e) - mg\cos(\gamma) \\ \dot{\theta} &= q, \\ I_{yy}\dot{q} &= \frac{1}{2}\rho V^2 S c C_M(\alpha, q, \delta_e) + T\ell_z \end{aligned} \tag{4}$$

where T is the power plant thrust, $\alpha = \theta - \gamma$ is the angle of attack, V is the aircraft airspeed, m is the aircraft mass, γ is the angle of flight, θ is the pitch angle, q is the pitch rotational speed, δ_e is the angle of deflection of the elevator, C_D, C_L and C_M are the aerodynamic coefficients drag, lift and pitching moment, respectively. Finally, ρ is the air density, S is the wing area, g is the gravity acceleration constant, I_{yy} is the inertia moment around the lateral axis, c is the wing mean aerodynamic chord, and ℓ_z is the vertical distance from the longitudinal axis of the power plant and the aircraft center of gravity.

It is assumed that the aerodynamic coefficients have the following structure [28]

$$\begin{aligned} C_D(\alpha) &= C_{D_0} + C_{D_1}\alpha + C_{D_2}\alpha^2, \\ C_L(\alpha, q, \delta_e) &= C_{L_0} + C_{L_1}\alpha + C_{L_q}\frac{qc}{2V} + C_{L_e}\delta_e \\ C_M(\alpha, q, \delta_e) &= C_{M_0} + C_{M_1}\alpha + C_{M_q}\frac{qc}{2V} + C_{M_e}\delta_e \end{aligned} \tag{5}$$

with $C_{D_0}, C_{D_1}, C_{D_2}, C_{L_0}, C_{L_1}, C_{L_q}, C_{L_e}, C_{M_0}, C_{M_1}, C_{M_q}$ and C_{M_e} are coefficients obtained from the CMARC software. Cmarc is a inviscid fluid flow analysis program of the type known as a low-order panel method. In order to obtain the aircraft dynamic modes it is necessary to linearize the nonlinear model in (4) around an equilibrium (trim) point [28]. The equilibrium point of (4) is defined by $\bar{V}$, $\bar{\alpha} = \bar{\theta} - \bar{\gamma}$, $\bar{\theta}$, $\bar{q}$, $\bar{T}$ and $\bar{\delta}_e$ such that

$$\begin{aligned} 0 &= \bar{T}\cos(\bar{\alpha}) - \frac{1}{2}\rho\bar{V}^2 S C_D(\bar{\alpha}) - mg\sin(\bar{\gamma}), \\ 0 &= -\bar{T}\sin(\bar{\alpha}) + \frac{1}{2}\rho\bar{V}^2 S C_L(\bar{\alpha}, \bar{q}, \bar{\delta}_e) - mg\cos(\bar{\gamma}) \\ 0 &= \bar{q}, \\ 0 &= \frac{1}{2}\rho\bar{V}^2 S c C_M(\bar{\alpha}, \bar{q}, \bar{\delta}_e) + \bar{T}\ell_z \end{aligned} \tag{6}$$

To obtain an equilibrium point, the airspeed and the flight path angle are fixed at $\bar{V} = 15$m/s, $\bar{\gamma} = 0$ rad, respectively. Hence, there are three equations and three unknown variables $\bar{T}$, $\bar{\alpha}$ and $\bar{\delta}_e$. Defining

$$x = \begin{bmatrix} V & \gamma & \theta & q \end{bmatrix}^\top, \; u = \begin{bmatrix} T & \delta_e \end{bmatrix}^\top$$

the aircraft longitudinal dynamics (4) can be written in compact form as follows

$$\dot{x} = F(x, u) \tag{7}$$

The linearized model of (7) around the equilibrium point

$$(\bar{x}, \bar{u}) = \{\bar{x}, \bar{u} \mid F(\bar{x}, \bar{u}) = 0\}$$

is given by

$$\dot{x}_\delta = A x_\delta + B u_\delta,$$

with

$$A = \left.\frac{\partial F}{\partial x}\right|_{x=\bar{x}, u=\bar{u}}, \; B = \left.\frac{\partial F}{\partial u}\right|_{x=\bar{x}, u=\bar{u}}$$

and (x_δ, u_δ) the approximation of (x, u) in the neighborhood of $(\bar{x}, \bar{u})$. The aircraft modes of motion are determined by the eigenvalues of the matrix A. The matrix A summarizes the aircraft dynamics characteristics with the control inputs fixed at the value corresponding to the equilibrium point. The eigenvalues have the following form [28]

$$\lambda_{1,2} := -a \pm bi, \; \lambda_{3,4} := -c \pm di$$

from where the frequencies and the damping coefficients of the modes of motion can be obtained as follows

$$\begin{aligned} \omega_{n_1} &= \sqrt{a^2 + b^2}, \; \zeta_{n_1} = \frac{a}{\omega_{n_1}}, \\ \omega_{n_2} &= \sqrt{c^2 + d^2}, \; \zeta_{n_2} = \frac{c}{\omega_{n_2}} \end{aligned} \tag{8}$$

If $\omega_{n_1} > \omega_{n_2}$ then ω_{n_1} defines the frequency of the short- period and ω_{n_2} is the frequency of the long-period (phugoide). The short and long period values as well as the corresponding

damping coefficients can be used to characterize the aircraft flying qualities as proposed in [5].
In this framework, the optimization problem is defined as follows. Let

$$\vec{x} \quad = \quad \{h_{wn}, \Lambda_{wn}, i_C, b_C, \Lambda_C\}$$

be the vector of decision variables. Consider the following optimization objectives

$$f_1(\vec{x}) = \max(\omega_{n_1}, \omega_{n_2}), \quad f_2(\vec{x}) = -T_{2ph} \tag{9}$$

where

$$T_{2ph} = \frac{\ln(2)}{-\zeta_{ph}\omega_{nph}} \tag{10}$$

with ζ_{ph} the damping coefficient for the phugoid or long-period oscillation mode. The value of T_{2ph} characterizes the time it would take for an unstable aircraft to double the amplitude of the oscillations after being disturbed. For unstable aircraft, this value is positive. For a stable aircraft, the value of T_{2ph} is negative, and its numerical value is the time it takes the aircraft to halve the oscillation amplitude after a disturbance. The proposed design objectives try to find a compromise between the two motion modes of an aircraft.
Now, we are in position to establish the optimization problem which in this case takes the following form

$$\min\{f_1(\vec{x}), f_2(\vec{x})\} \tag{11}$$

constraint to

$$\begin{array}{ll} 0.1m \leq h_{wn} \leq 0.2m, & 0.0^o \leq \Lambda_{wn} \leq 45^o, \\ -10^o \leq i_C \leq +5^o, & 0.1m \leq b_C \leq 0.25m, \\ -30^o \leq \Lambda_C \leq +30^o & \end{array} \tag{12}$$

Once we have defined the objective functions to optimize (minimize, in this case), we must implement a multi-objective optimization method. In the literature, various optimization methods range from those using mathematical programming techniques [21] to those using evolutive meta-heuristic algorithms [6]. For the latter case, we can distinguish algorithms based on genetic algorithms [8], evolutionary strategies [33], clusters of particles in [30], Differential Evolution [32] , among others. This article uses a multi-objective evolutionary algorithm, called MODE -LD + SS, proposed in [1]. This algorithm makes use of meta-heuristics called Differential Evolution. The multi-objective optimization process will start with a population of 50 individuals. Since Evolutionary Algorithms are stochastic processes, several runs are needed for several generations. In these runs, possible solutions were accumulated in the order of a thousand. The solutions found from different runs were processed to determine those that are globally non-dominated and thus approximate the Pareto front solution representing the problem. The aerodynamic properties of all tested geometric configurations are obtained during the optimization process using the CFD program CMARC. The following pseudocode illustrates the workflow.

Algorithm 1 Workflow

```
Require: 50 aircraft configurations.
Ensure: Design variables on (12).
for $i = 50, ..., 1000$ do
    Aerodynamic coeffients determination using CMARC.
    Objective functions computation.
end for
Search for non-dominated solutions.
```

Figure 2 shows the flying wing representation to be used in the CMARC software.

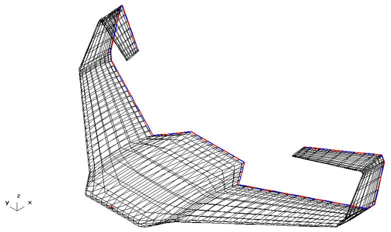

Figure 2: Aircraft panels model to be used by CMARC

4 RESULTS

This section presents the results obtained from the multi-objective optimization process. Figure 3 shows the approximation of the Pareto front obtained for this problem; the front has approximately 52 non-dominated solutions. From the Pareto front, it can be observed that there is indeed a con-

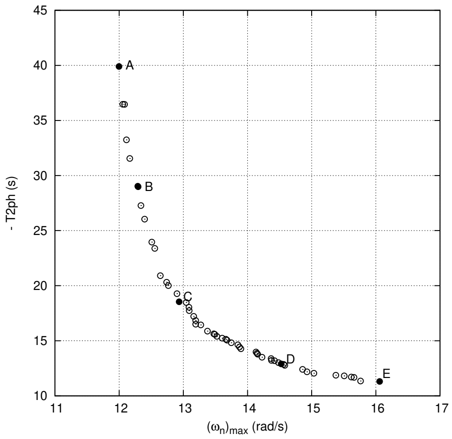

Figure 3: Multiple solutions at the Pareto front.

flict between the two objectives considered in this problem. Additionally, it can be verified that all obtained aircraft configurations are stable. Each solution represents a combination of different short and long modes. Five solutions are identified in the Pareto front of Figure 3, labeled A, B, C, D, and E, respectively. Solution A has the minimum value of the first objective function and the maximal value for the second objective function. Thus, the corresponding aircraft will have a lightly damped long-period mode and a minor short-period frequency. The solution E has antisymmetric characteristics, a better damped long-period, and a more significant short-period frequency. Solution C is in the middle part of the Pareto front approximation, and it could be considered the one that represents the best compromise between the two objective functions. In order to compare the different sets of geometric parameters obtained in each of the five selected solutions, Table 2 shows the respective values for the geometric variables. The geometry of the selected solutions is shown in figure 4. From the analysis of the results, as well as the tabulated geometric characteristics for the five selected configurations, it can be observed that the parameters defined as variables in the optimization problem significantly influence the objective functions. Consequently, this influence reflects on the flying qualities of the UAV platform, defined in terms of the frequencies of the aircraft's motion modes.

Solution	h_{wn}	Λ_{wn}	i_C	b_C	Λ_C
A	0.1000	37.6814	3.0417	0.1000	20.9189
B	0.1336	32.8245	4.9212	0.1000	26.6503
C	0.1220	45.0000	5.0000	0.1423	-13.2555
D	0.1026	0.0000	5.0000	0.2500	-30.0000
E	0.1870	0.0000	4.3187	0.2408	-27.5345

Table 2: Design variables for the solutions A,B,C,D, and E on the Pareto front

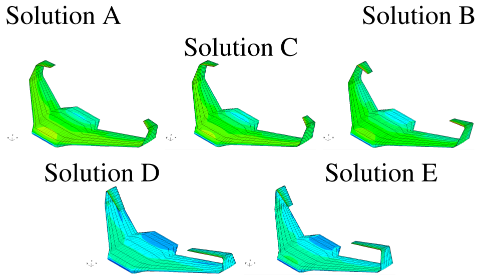

Figure 4: Selected solutions from the Pareto front.

It is difficult to identify trends in the design variables, such as the height of the winglet h_{wn} and the incidence of the C-wing tip i_C, because it is the combination and interaction with other variables that generate the aircraft's flying qualities. Some characteristics of the solutions can be described as follows. Note that winglet swept Λ_{wn} goes towards the upper limit in the solution at the central part of the Pareto front, solution C. For the C-wing swept Λ_C, it is observed that positive values are associated with solutions towards the left part of the Pareto front, solutions A and B, while in the central and right part of the front, the solutions take a negative values. Moreover, small values for the C-wing span b_C are associated with solutions on the left side and up to the central part of the Pareto front, while the solutions on the right side take large C-wing span values near the upper limit.

Concerning the compromise between the objective functions, it is observed that the solutions with the lowest value of the short period consequently have a more extended long period, reaching times of up to 40 seconds to damp to half the oscillation after a disturbance. The multi-objective analysis allows designers to identify solutions and to identify for each of them the values of the objective functions. From the solution set, the control system designer will decide which would be the best choice regarding the bandwidth of the sensors, actuators, and the processing device installed in the UAV control system. Since all aircraft configurations are stable, solution C will be preferred by the control system designer, while solution A will be preferred by a human pilot.

5 CONCLUSIONS

This work presents the application of a multi-objective design methodology to the practical case of the conceptual design of a C-wing tip for a flying wing UAV platform. Five design variables are considered to explore the relationship between the aircraft's geometric characteristics and its dynamic characteristics expressed in terms of flying qualities expressed as the modes of longitudinal flight dynamics. The selection of the solutions that represent the best compromise between the different objectives was based on Pareto dominance. Since it is difficult to identify trends in design variables, such as the height of the winglet h_{wn} and the incidence of the C-wing tip i_C, because it is the combination and interaction with other variables that generate the aircraft flying qualities, it is proposed as future work to develop a factorial statistical experiment design to calibrate the design variables. Consequently, the design of experiments will provide information to establish the most precise optimization area through response surfaces.

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