Actuator Fault Detection and Diagnosis for Quadrotors

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ABSTRACT

This paper presents a method for fault detection and diagnosis of actuator loss of effectiveness for a quadrotor helicopter. This paper not only considers the detection of the actuator loss of effectiveness faults, but also addresses the diagnosis of the faults. The detection and estimation of the faults are performed by the Augmented Extended Kalman Filter. The faults are modelled as random walk processes and are treated as additional states which makes the fault estimation unbiased. The estimated faults can be further used for Fault Tolerant Control. Simulation both without and with the occurrence of the actuator faults demonstrates the effectiveness of the proposed approach.

1 INTRODUCTION

Fault Detection and Diagnosis (FDD) is essential for achieving a successful control system reconfiguration [1]. Many techniques have been proposed especially for sensor and/or actuator faults [2, 3, 4]. Among these, model-based techniques have been widely studied and even successfully applied to several industrial areas [3, 5]. Many fault detection filters have been proposed to tackle this problem [3, 6].

Kalman Filter (KF) [7] is an optimal filter for linear system when the system is working nominally. If there are model uncertainties or faults, the state estimate of the filter will deviate from the real state. Furthermore, in practice, the system are always nonlinear if no assumptions are made. Therefore, the KF should be extended to nonlinear filter in order to cope with nonlinearities in the system. One way is to use the Extended Kalman Filter (EKF) [8], which requires the calculation of Jacobian matrix of the system matrix.

In this paper, the loss of effectiveness fault of a quadrotor helicopter is considered. In [9], the faults were detected by using the innovation of the KF. However, how to estimate the faults which are essential for Fault Tolerant Control (FTC) was not addressed. This paper uses the same fault detection technique and also addresses the fault diagnosis. The faults will be estimated by the Augmented Extended Kalman Filter (AEKF). The AEKF models the faults as additional states. The estimated faults can provide a more accurate information for the FTC.

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The structure of the paper is as follows: Section 2 will present some preliminaries related to this paper. Section 3 will present the mathematical model of the quadrotor helicopter which is used in this paper. The modelling of the actuator faults is also introduced. The FDD of the actuator loss of effectiveness fault is presented in Section 4. The result without and with the fault are shown in Section 5 while Section 6 concludes the paper.

2 PRELIMINARIES

This section will present the problem which will be coped with in this paper.

2.1 Nonlineas system with input faults

Consider the nonlinear stochastic system with input faults in the discrete form

$$\begin{cases} x_{k+1} = f(x_k, u_k, k) + w_k + f_{i,k} & (1)\\ y_{k+1} = h(x_{k+1}, k+1) + v_{k+1} & (2) \end{cases}$$

where $x_{k+1} \in \mathbb{R}^n$ represents the system states, $u_k \in \mathbb{R}^m$ the control input, $y_{k+1} \in \mathbb{R}^p$ the measurement. w_k and v_k are the process noise and measurement noise vector respectively. The function $f_{i,k}$ represents input faults respectively. It is assumed that the noise vectors w_k and v_{k+1} are zero-mean and

$$E\{w_k\} = 0\ ,\ E\{w_k w_\tau^T\} = Q_k\ \delta_{k\tau} \quad (3)$$

$$E\{v_k\} = 0\ ,\ E\{v_k v_\tau^T\} = R_k\ \delta_{k\tau} \quad (4)$$

$$E\{w_k v_k^T\} = 0 \quad (5)$$

where the $\delta_{k\tau}$ denotes the Kronecker delta function, Q_k and R_k are the covariance matrix of the process noise and measurement noise respectively.

2.2 Extended Kalman Filter

This section presents the EKF [8, 10] which is used for the state estimation of the nonlinear system. The EKF, which is a form of the KF extended to nonlinear systems, can be applied to estimate the states. The five standard steps for an EKF are as follows:

1. One step ahead prediction

$$\hat{x}(k+1|k) = \hat{x}(k|k) + \int_k^{k+1} f(x(t), u(t), t)dt \quad (6)$$

2. Covariance matrix of the state prediction error

$$P(k+1|k) = \Phi(k,\tau)P(k|k)\Phi^T(k,\tau) + Q_d(k) \quad (7)$$

where $\Phi(k,\tau)$ is calculated as follows:

$$\Phi(k,\tau) = e^{F(k)\Delta t} = \sum_n^\infty \frac{F_k^n(\Delta t)^n}{n!} \quad (8)$$

where $F(k)$ is the linearized matrix computed as follows:

$$F(k) = \left.\frac{\partial f(x(t),u(t),t)}{\partial x(t)}\right|_{x=\hat{x}(k|k)} \quad (9)$$

The $Q_d(k)$ is calculated as follows:

$$\begin{aligned} &Q_d(k+1|k) \\ &= E\{w_d(k)w_d^T(k)\} \\ &= \int_{t_k}^{t_{k+1}} \Phi(t_{k+1},\tau)G_kQ_kG_k^T\Phi^T(t_{k+1},\tau)d\tau \\ &\approx \sum_m^\infty \sum_n^\infty \frac{F_k^m}{m!}G_kQ_kG_k^T\frac{F_k^n}{n!}\frac{(\Delta_t)^{n+m+1}}{(n+m+1)!} \quad (10) \end{aligned}$$

However, there is a commonly used approximation for $Q_d(k)$ which is calculated as follows:

$$Q_d(k) = \Gamma(k)Q(k)\Gamma(k)^T \quad (11)$$

where $\Gamma(k)$ is calculated as follows:

$$\Gamma(k) = \left(\int_{k-1}^{k} \Phi(k)\Delta t\right)G(k) \quad (12)$$

3. Compute the Kalman gain
The optimal Kalman gain is calculated by the following equation:

$$K(k+1) = P(k+1|k)H^T(k+1)V^{-1}(k+1) \quad (13)$$

where $V(k+1)$ is calculated by

$$V(k+1) = H(k+1)P(k+1|k)H^T(k+1) + R(k+1) \quad (14)$$

where $H(k+1)$ is the linearized matrix of the measurement matrix

$$H(k) = \left.\frac{\partial h(x(t),u(t),t)}{\partial x(t)}\right|_{x=\hat{x}(k|k)} \quad (15)$$

4. Measurement update step:

$$\begin{aligned} \hat{x}(k+1|k+1) = \hat{x}(k+1|k) \\ + K(k+1)\gamma(k+1) \quad (16) \end{aligned}$$

where $\gamma(k)$ is the innovation of the EKF, which is computed as

$$\gamma(k+1) = \big(y(k+1) - H(k+1)\hat{x}(k+1|k)\big) \quad (17)$$

5. Update the covariance matrix of the state estimation error matrix

$$\begin{aligned} P(k+1|k+1) = (I - K(k+1)H(k+1))P(k+1|k) \\ \times(I - K(k+1)H(k+1))^T \\ +K(k+1)R(k+1)K^T(k+1) \quad (18) \end{aligned}$$

Since the EKF is a linearized form of the KF, problems may occur when the linearisation error is large. Note the EKF is only convergent when the initial states are close enough to the real states.

3 MATHEMATICAL MODEL AND FAULT MODEL OF THE QUADROTOR

This section will present the mathematical model of the quadrotor and build the fault model for the actuator faults of this quadrotor. Note the model used here is the same with [9]. The model used is the Qball-X4 model.

3.1 Model of the quadrotor

In this paper, the objective is to detect and estimate the faults of the quadrotor actuators. The model of a quadrotor can be described as follows [11]:

$$\dot{x} = v_x \quad (19)$$
$$\dot{y} = v_y \quad (20)$$
$$\dot{z} = v_z \quad (21)$$
$$m\dot{v}_x = U(\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi) \quad (22)$$
$$m\dot{v}_y = U(\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi) \quad (23)$$
$$m\dot{v}_z = U(\cos\phi\cos\theta) - mg \quad (24)$$
$$J_1\ddot{\phi} = \tau_\phi \quad (25)$$
$$J_2\ddot{\theta} = \tau_\theta \quad (26)$$
$$J_3\ddot{\psi} = \tau_\psi \quad (27)$$

where x, y and z are the positions of the quadrotor, v_x, v_y and v_z are the velocity components of the quadrotor. m is the mass of the quadrotor and g is the gravity constant. ϕ, θ and ψ are the attitude angles of the quadrotor which are roll, pitch and yaw angle respectively. J_1, J_2 and J_3 are the moments of inertia of the quadrotor. U is the total lift and τ_ϕ, τ_ϕ and τ_ϕ are the torques along the directions of the ϕ, θ and ψ angles respectively.

It is assumed that all the states mentioned above are measured. However, the measurements are corrupted by noise. The measurement model is as follows:

$$x_m = x + \nu_x \tag{28}$$
$$y_m = y + \nu_y \tag{29}$$
$$z_m = z + \nu_z \tag{30}$$
$$v_{xm} = v_x + \nu_{vx} \tag{31}$$
$$v_{ym} = v_y + \nu_{vy} \tag{32}$$
$$v_{zm} = v_z + \nu_{vz} \tag{33}$$
$$\phi_m = \phi + \nu_\phi \tag{34}$$
$$\theta_m = \theta + \nu_\theta \tag{35}$$
$$\psi_m = \psi + \nu_\psi \tag{36}$$

where $\nu = [\nu_x\ \nu_y\ \nu_z\ \nu_{vx}\ \nu_{vy}\ \nu_{vz}\ \nu_\phi\ \nu_\theta\ \nu_\psi\ ]^T$ are the noises in the measurements.

3.2 Model of the actuator

This subsection will introduce the model of the actuator and the fault modelling of the actuator faults. The actuator dynamics are as follows:

$$U = \frac{K\omega}{s+\omega}u \tag{37}$$

where K is a gain factor and ω is a parameter of the actuator. The input of the actuator is the PWM. Therefore, the model of the actuator with the PWM input is as follows:

$$\dot{U} = -\omega U + K\omega\,[1\ 1\ 1\ 1]\begin{bmatrix} PWM_{1t} \\ PWM_{2t} \\ PWM_{3t} \\ PWM_{4t} \end{bmatrix} \tag{38}$$

where PWM_{it}, $i = 1, 2, 3, 4$ are the theoretical four PWM inputs respectively. Also note that the initial PWM outputs are the minimum throttle value 0.05 [9]. Therefore, the PWM_{it} should be changed to

$$PWM_{it} = PWM_{ir} - 0.05,\ i = 1, 2, 3, 4 \tag{39}$$

where $PWM_{ir} \in (0.05, 0.1)$ are the real PWM inputs to the actuator.

3.3 Actuator fault modelling

This subsection will consider the modelling of the actuator faults of the quadrotor. The fault considered in this paper is partial loss of control effectiveness. Let l_i, $i = 1, 2, 3, 4$ denote the loss of effectiveness. Then the model of the actuator is

$$\begin{aligned} \dot{U} &= -\omega U + K\omega\,[1-l_1\ 1-l_2\ 1-l_3\ 1-l_4]\begin{bmatrix} PWM_{1t} \\ PWM_{2t} \\ PWM_{3t} \\ PWM_{4t} \end{bmatrix} \\ &= -\omega U + K\omega\,[1\ 1\ 1\ 1]\begin{bmatrix} PWM_{1t} \\ PWM_{2t} \\ PWM_{3t} \\ PWM_{4t} \end{bmatrix} \\ &\quad - K\omega[PWM_{1r}, PWM_{2r}, PWM_{3r}, PWM_{4r}]\begin{bmatrix} l_1 \\ l_2 \\ l_3 \\ l_4 \end{bmatrix} \end{aligned} \tag{40}$$

Now the actuator fault model has been established, the objective of this paper is to detect and diagnose the faults, which will be introduced in the following section.

4 Fault Detection and Diagnosis of the Actuator Faults

This section will present the method for the detection and diagnosis of the loss of effectiveness fault for the quadrotor. In [9], the actuator faults can be detected using the innovation of the EKF. However, the fault reconstruction is not considered which is essential for fault tolerant control. The fault estimation is also beneficial for the diagnosis of the system. In this section, the fault detection will be first introduced.

The fault detection is based on the innovation of the filter

$$\gamma(k+1) = \big(H(k+1)x(k+1) - H(k+1)\hat{x}(k+1|k)\big) \tag{41}$$

In order to detect the faults, a threshold T_0 needs to be defined. The fault detection logic is

$$\gamma(k+1) = \begin{cases} \geq T_0 \ \Rightarrow \ \text{trigger the fault alarm} \\ < \ T_0 \ \Rightarrow \ \text{No faults} \end{cases} \tag{42}$$

Since there are noises in the measurement, T_0 should be a number bigger than zero. One can also accumulate the innovation in a certain length of time window to detect the faults.

4.1 Augmented Extended Kalman Filter

In this paper, we propose to use the AEKF to estimate the faults in the actuators of the quadrotor. The AEKF is an augmented form of the EKF. The faults are augmented as additional states so the faults are also estimated by the AEKF.

To estimate the faults, we need to have a fault model. One way to model the dynamics of the time-varying input fault is to treat it as a random walk process[12, 13, 14]. This strategy is combined with the filter for the purpose of improving the state estimation performance in the presence of unknown input faults. In order to achieve this, a random signal w'_k with a covariance matrix Q'_k is introduced to the dynamic model of the filter

$$\dot{x}'(t) = \begin{bmatrix} \dot{x}(t) \\ \dot{f} \end{bmatrix} = \begin{bmatrix} f(x(t), u(t), t) \\ 0 \end{bmatrix} + \begin{bmatrix} w(t) \\ w'_k(t) \end{bmatrix} \quad (43)$$

where f is the vector for the loss of effectiveness fault. Since there are four PWM inputs, the vector of the faults is denoted as

$$f = [f_1 \ f_2 \ f_3 \ f_4]^T \quad (44)$$

By augmenting the faults as the state vector, the faults can be estimated in an unbiased sense. The value of the f_i, $i = 1, 2, 3, 4$ is the estimated value of l_i. Therefore, the estimation of the loss of effectiveness fault is achieved.

5 Simulation Results

In this section, the approach proposed in the previous section will be applied to the quadrotor actuator fault diagnosis. The baseline controller is a PID controller which controls the altitude of the quadrotor. The position x and y are not controlled.

In order to show the performance of the approach, the situation with and without actuator faults will be shown.

5.1 FDD when there are no faults

In this simulation, there are no actuator faults. All the PWM inputs are normal. This means that $l_1 = l_2 = l_3 = l_4 = 0$. The results of the actuator fault diagnosis using the AEKF are shown in Figure 1 - Figure 5.

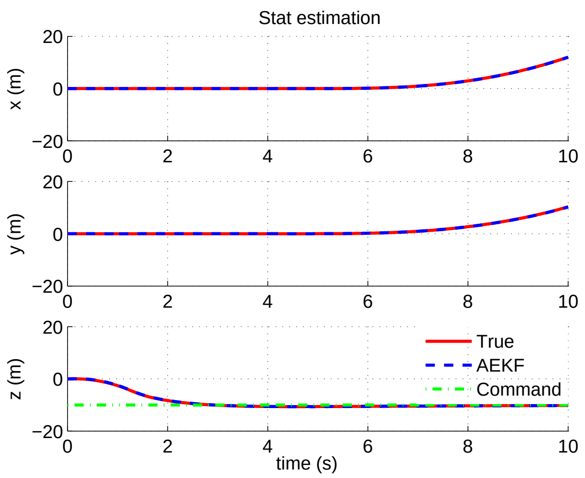

Figure 1: State estimation of the KF without faults

Figure 1 shows the controller response. The controller can track the command of the altitude which is given in green line. The red lines denote the true states of the quadrotor while the blue dotted lines represent the estimation of the AEKF. Since there is only a controller for the altitude, the position x and y are not controlled.

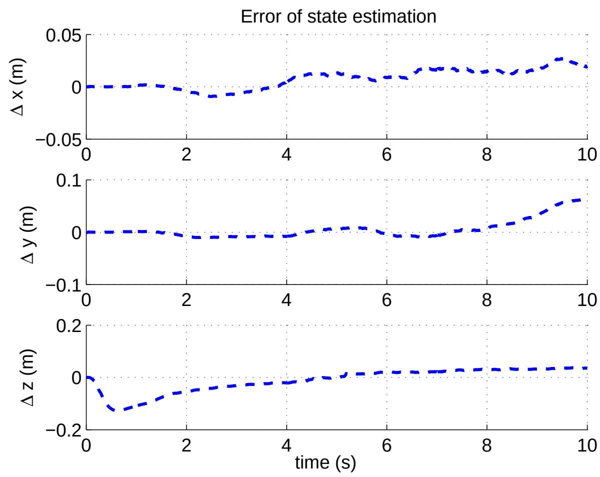

Figure 2: State estimation of the KF without faults

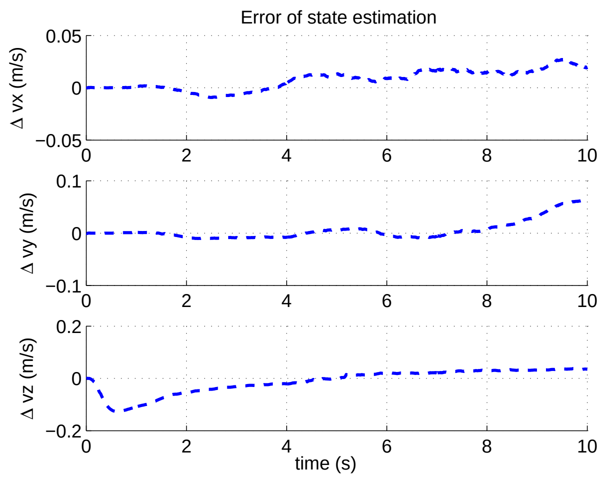

Figure 3: State estimation of the AKF without faults

The state estimation error of the position vector, velocity vector and attitude angles using the AEKF are shown in Figure 2, Figure 3 and Figure 4 respectively. As can be seen, all the estimation error are close to zero-mean. This demonstrates the state estimation ability of the AEKF.

The fault estimation using the AEKF is shown in Figure 5. As can be seen, the estimation requires almost one seconds to converge to the true value. After that, the estimation error is zero-mean.

5.2 FDD when there are actuator faults

In this simulation, all the four PWM suffer from a loss of effectiveness fault. The magnitude of the fault is $0.1t$, and the limit for the loss of effectiveness factor is 0.5. The faults of two actuators related to the pitch axis are injected at $t = 5$ s whereas those of two actuators related to the roll axis are injected at $t = 6$ s.

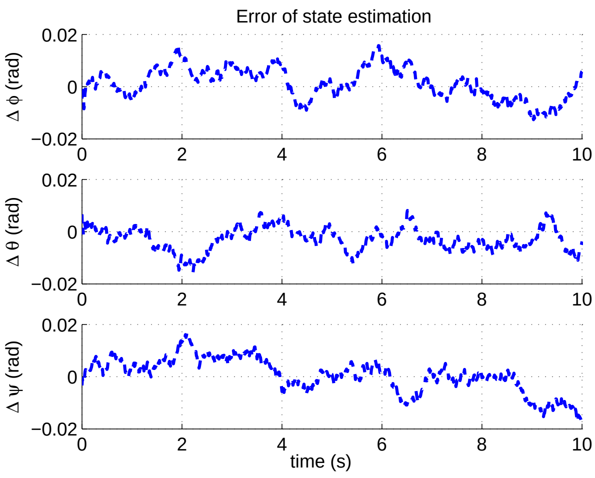

Figure 4: State estimation of the AKF without faults

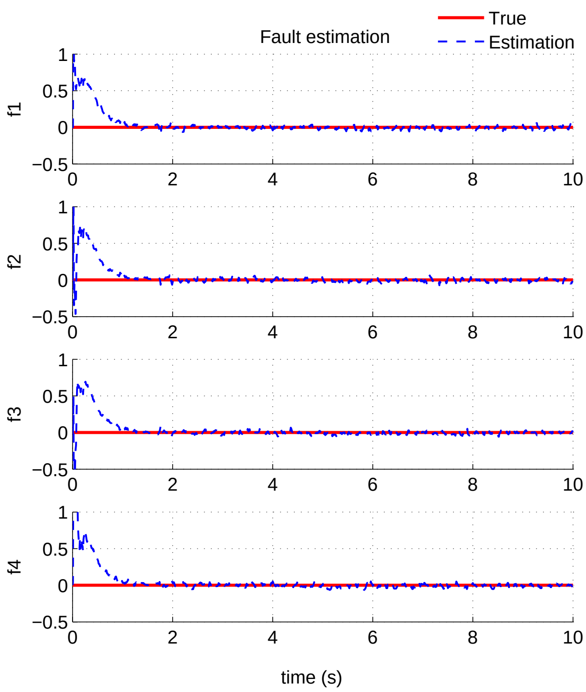

Figure 5: Fault estimation of the AKF without faults

Note the faults injected here shows a loss of effectiveness fault. The result is shown in Figure 6.

The state estimation errors using the AEKF are similar to the situation when there are no actuator faults. Therefore, they are not shown here. The controller response is also similar to Figure 1. The fault estimation using the AEKF is given in Figure 6. As can be seen from the figure, the loss of effectiveness faults of the four actuators can be estimated correctly. However, there is a time delay between the true faults and the estimated faults. This can be improved by changing the mag-

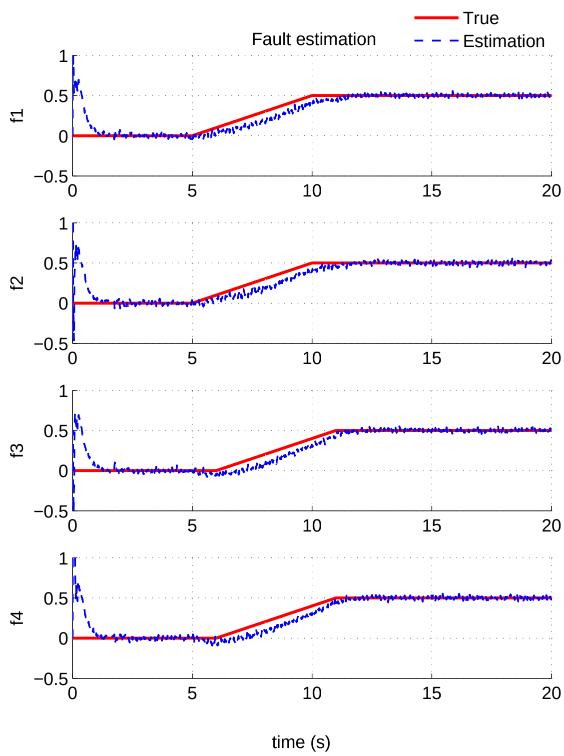

Figure 6: State estimation of the KF with faults

nitude of the random signal $w'(k)$ in Equation 43. Increasing the magnitude can reduce the time delay. However, the noise magnitude of the fault estimation will be bigger.

6 Conclusions

This paper addresses the FDD of the actuator loss of effectiveness fault for a quadrotor helicopter. The model of the quadrotor was built and the modeling of the faults was also introduced. The detection of the faults was addressed by the innovation of the filter. The diagnosis of the faults was addressed by modelling the faults as a random walk process. Therefore, the estimation of the fault achieved an unbiased estimate by treating the faults as additional states. The estimated faults can be used for FTC. Two situation was simulated to show the performance of the proposed approach. The controller was able to follow the command successfully both with and without the faults. The faults were also estimated in an unbiased sense, which verified the effectiveness of the approach.

Future work includes designing a controller for the position including x and y. Furthermore, the implementation of the proposed approach on a real quadrotor should also be carried out.

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