the transitions from subcritical Reynolds numbers to supercritical ones and back to the subcritical Reynolds numbers run in different ways. This phenomenon is named as the aerodynamic hysteresis [14]. Almost all aerodynamic characteristics of micro-UAV become highly nonstationary and they depend also from dynamics of changes for Reynolds number values. Moreover large UAV angular velocities about Z axis can cause critical angle of attack value and stalling of the UAV. Various airfoils have different kinds of $C_L(\alpha)$ function with more steep or more smooth function value changes for $\alpha > \alpha_{CR}$ i.e. for angle of attack values exceeded critical ones. As a rule airfoil with relatively large radius of it forebody have more smooth stall characteristics while airfoil with small forebody radius have more steep characteristics.

We can see now that airfoil have an influence on non-stationary aerodynamic characteristics of UAV because of dependence between these characteristics and Reynolds number.

Thus, the reasonable selection of airfoil for micro-UAV must be some compromise between desired airfoil properties for each UAV flight regime and actual characteristics for the selected airfoil.

The design philosophy suggested in our paper is based on usage of some collection of alternative airfoils with known aerodynamic characteristics to choose a reasonable alternative contained in the collection. A synthesis of new airfoils additional to the alternatives of the collection is a separate problem which is not considered in this paper.

3 Design philosophy based on multitask approach for reasonable airfoil selection in regard to micro-UAVs

Thus, as it was stated above, we need to solve the problem of compromise airfoil selection for the wing of a micro-UAV. It is necessary to make this selection for some range of flight regimes which differ one from another with airspeed values. Let us consider this selection problem in wider statement. We will suppose that airspeed is only one element from a set of flight tasks and application conditions for designed micro-UAV.

A problem of adequate representation for the source set of flight tasks and application conditions are very important for the designed micro-UAV as well as for its components especially for airfoil. This problem has great significance for micro-UAVs both for their theoretical issues and applications. A solution of this problem determines immediately requirements specification for the designed micro-UAV. In addition it predetermines optimization approaches and techniques used for micro-UAV design.

Two different approaches are used in contemporary design activity. First of them is based on replacement (by means of aggregation technique) of source set *X* of flight tasks and application conditions by another set X'. The X' set has fewer elements than the *X* set. For marginal case of the X' set it consists of only one element *x** which is some typical task named usually as 'design task' or 'nominal task' (see Figure 1).

However this approach causes the problem of appropriate selection for the design task *x** to represent sufficiently the source set *X*.

An alternative approach suggested and developed in the USSR in the middle of 1960s is presented in [11], [12], and [13].

This multitask approach takes into account the set of flight tasks and application conditions through introducing of the external set *X*. We choose values of design parameters for UAV according to the multitask approach by means of appropriate optimization problem solving.

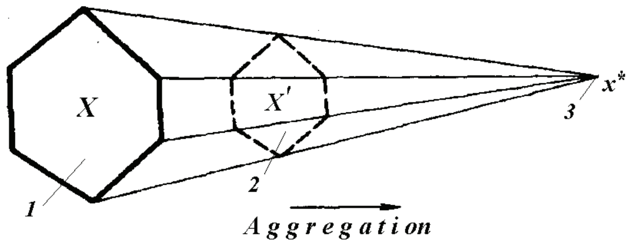

Figure 1: Replacement of the external set *X* by the design task x^*: 1 – external set *X*; 2 – aggregated external set X'; 3 – design task x^*

This problem involves some unified operation criterion [12], [13]

(1) $F\left[X, y, u(t)\right]$,

required to be minimized or maximized subject to UAV design parameters; here ***y*** is vector of design parameters and ***u(t)*** is vector of control law parameters for UAV.

Following the aggregated approach [13] we take into consideration tasks and application conditions as some design task $x^* \in X$ in a mathematical model of optimal design. This design task *x** is usually the main element of requirements to the developed UAV and it is derived by means of design task analysis as well as analysis of the source set of flight tasks and conditions of their accomplishment.

As an example of design task x^* for micro-UAV we can specify a flight operation to search some small surface object for prescribed search range *D* with predetermined UAV payload. A design task in airfoil case can be stated as a cruising flight with some prescribed airspeed ***U***. The influence of airspeed value on reasonable airfoil selection for wing of micro-UAV was discussed above.

According to this approach the optimal design process consists in selection of some alternative micro-UAV version, which is most effective for the prescribed design task. However in real flight conditions our micro-UAV has to be capable to run not only this design task but a set of other flight tasks. Therefore parameter values of the UAV must ensure some design compromise to run efficiently every flight task of the set although these values are not possibly the best for any task.

On Figure 2 we can see how concepts introduced above relate to such important micro-UAV design element as wing airfoil. This example uses a one-dimensional continuous set of tasks $U = \left[U_0, U_1\right]$ which is the range of UAV airspeeds. Figure 2 shows us that the first airfoil version (Profile 1) is most efficient for the design task U_r^*, but this airfoil has large drag losses (they marked with hatching an Figure 2) for off-nominal values of airspeed, therefore the Profile 2 airfoil is preferable in regard to discussed case.

There are no doubts that it is necessary to consider quantitatively the whole set of flight tasks and application conditions for the designed micro-UAV and its elements. If we build the optimal design model taking into account the diversity of prescribed tasks then we can avoid considerable losses of effectiveness for developed UAV. This problem can be solved using the concept of external set *X* and appropriate optimization algorithms.

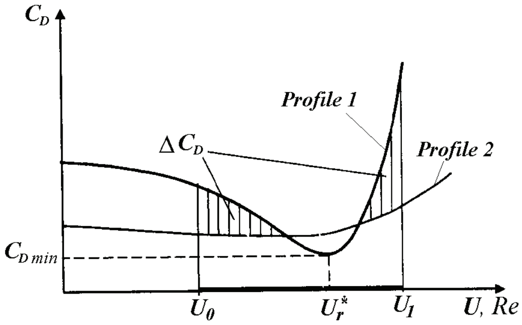

Figure 2: A drag losses for the Profile 1airfoil due to off-nominal values of airspeed

However such approach complicates obviously the optimization model for micro-UAV design parameters as well as it enhances efforts needed to solve optimal design problem. The question is natural about reasonability of this complication as compared with the conventional optimal design problem. It was shown in [11], [12], and [13] as an answer to this question that deriving of the design task x^* basing only on the information about external set *X* leads to a considerable error in UAV effectiveness estimation.

4 Problem of multitask optimization and a way to solve it

Thus, we can see that a choice of optimal parameters can be represented as an optimization problem for some simple scalar multitask system with external set *X*, set of strategies *Y*, and strategies $A=\{y_i \in Y\}$, $i=1,\ldots,m$.

An efficiency index for this multitask system can be specified in two ways according to [12] as some efficiency function in regard to the optimal design problem for a system of UAV airfoils or wings.

First of all, the efficiency function in case of *integrated multitask system* can be stated as

$$(2)\quad F(X, A, E(x)) = \int_X p(x)\; G(x, y_i)\; dx$$

or

$$(3)\quad F(X, A, E(x)) = \int_X p(x)(G(x, y_i) - \overline{G}(x))dx$$

Variable *F(X, A, E(x))* defined by Equation 2 corresponds to the mean value of the functional $G(x, y_i)$. This value is related to a single task from the external set *X* which is a region of flight tasks in the considered problem. The optimization problem using this efficiency index is equivalent to the well-knows unification problem [11], [12], and [13]. Solving this problem it is possible to derive optimal values for design parameters of the system of UAV airfoils if we know the *p(x)* function.

Variable *F(X, A, E(x))* defined by Equation 3 corresponds to the value of absolute deviation of the functional $G(x, y_i)$ from the value $\overline{G}(x)$ which this functional could possess for the UAV wing optimized in regard to the flight regime $x \in X$.

In the second case which is *guaranteed multitask system* one, the efficiency function for UAV system of airfoils/wings $A=\{y_i\}, i=1,\ldots,m$ can be stated as

$$(4)\quad F(X, A, E(x)) = \max_{x\in X} \rho(x, y_i),$$

where $E(x)$ is a *distribution function,* which connects airfoil alternatives with their reasonable usage regions for the predetermined external set *X*.

Here we introduce the functions

$$(5a)\quad \rho(x, y_i) = \frac{G(x, y)}{\overline{G}(x)}$$

or

$$(5b)\quad \rho(x, y_i) = \frac{G(x, y) - \overline{G}(x)}{\overline{G}(x)}$$

which define the nonoptimality degree of arbitrary UAV airfoil y_i for a flight regime (flight task) $x \in X$ in comparison with the airfoil optimized for the same regime $x \in X$.

Then efficiency function defined by Equation 4 is maximal nonoptimality degree for the UAV airfoil y_i in regard to the whole region of tasks *X*.

We can formulate now a general optimization problem named also as strategy optimization problem for described system of airfoils treated as multitask system.

The system of airfoils $A=\{y_i\}, i=1,\ldots,m$ is optimal for the set *X* of flight regimes $x \in X$ if:

1) the collection of airfoils $A=\{y_i\}, i=1,\ldots,m$ provides maintenance of the UAV for all flight regimes of *X*;
2) the efficiency function value is maximal (see Eq. 6) or minimal (see Eq. 7) for this collection, i.e.

$$(6)\quad F(X, \overline{A}, \overline{E}(x)) = \min_{A\subset Y, E(x)} \int_X p(x)\; G(x, y_i)\; dx$$

$$(7)\quad F(X, \overline{A}, \overline{E}(x)) = \min_{A\subset Y, E(x)} \int_X p(x)\; (G(x, y_i) - \overline{G}(x))\; dx$$

or the value of maximal nonoptimality degree is minimal for this system of airfoils

$$(8)\quad F(X, \overline{A}, \overline{E}(x)) = \min_{A\subset Y, E(x)} \max_{x\in X} \rho(x, y_i)$$

Let us notice that we can write expressions according to [12]

$$F(X, \overline{A}, \overline{E}(x)) = \min_{A\subset Y} \int_X p(x) \min_{i=1,\ldots,m} G(x, y_i)\; dx$$

$$F(X,\overline{A},\overline{E}(x)) = \min_{A\subset Y}\max_{x\in X}\min_{i=1,\dots,m}\rho(x,y_i)$$

due to specific nature of relationships described by Equations 2, 3, and 4.

We need to specify realization rate function *p(x)* for all flight regimes $x \in X$ to determine indices (6) and (7).

However, the efficiency index specified by Equation 8 allows us to derive optimal design parameters of the airfoil system $A=\{y_i\}, i=1,\dots,m$ without information about distribution of flight tasks in the region *X*. It is very convenient on early stages of UAV design process when it is especially important to take into account expected region of flight tasks. Optimization according to this efficiency index provides UAV maintenance for each flight regime of the region *X*. This approach ensures that nonoptimality degree described by Equation 5 do not exceed some limiting value obtained from solution of the problem defined by Equation 8.

The suggested optimality criteria is coincide with conventional minimality conditions if the external set *X* is reduced to one flight task, i.e. the UAV will operate only in one flight regime.

5 Solution example of multitask optimization problem for micro-UAV

We will discuss in this section a solution example for multitask micro-UAV optimization problem. Our goal is to select the most reasonable airfoil from the predetermined collection of airfoils with known aerodynamic characteristics. Lift to drag ratio $K = C_L / C_D$ is used here as an efficiency function *G(x)* together with ΔK as a value of relative loss for a wing with arbitrary airfoil $y_i \in \{y_i\}, i=1,\dots,m$ as applies to a flight regime $x \in X$ in comparison with the airfoil optimized for the same regime $x \in X$.

Airfoil 1	Eppler 61
Airfoil 2	Eppler 193
Airfoil 3	FS 60100
Airfoil 4	Göttingen 495

Table 1: Dominance of low speed flights (U = 8–12 m/sec)

Airfoil 1	G 532
Airfoil 2	N-81
Airfoil 3	NACA 4512
Airfoil 4	NACA 4212

Table 2: Dominance of middle speed flights (U = 15–20 m/sec)

Airfoil 1	NACA 19
Airfoil 2	NACA 20
Airfoil 3	NACA 15
Airfoil 4	NACA 17

Table 3: Dominance of large speed flights (U = 25–30 m/sec)

Example. A rational choice of the most suitable airfoil from the predetermined collection.

We have some predetermined collection of airfoils with known aerodynamic characteristics (see Tables 1, 2, and 3) as well as three versions of the weight function *p(x)* which is interpreted here as a quantity *N* of flights carried out with different airspeeds (see Figures 3, 4, and 5).

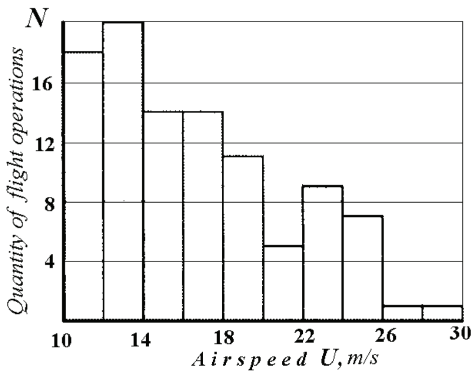

Figure 3

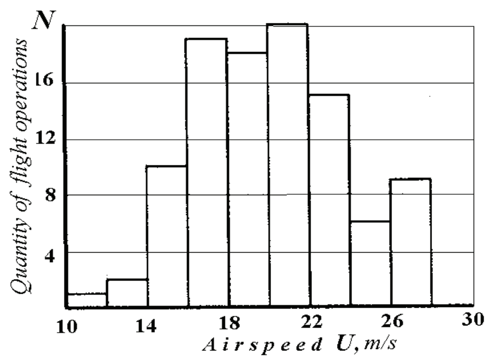

Figure 4

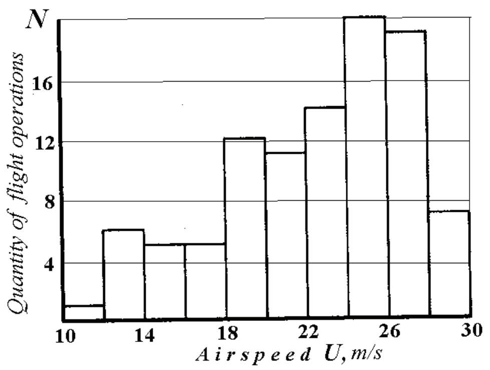

Figure 5

Aerodynamic characteristics for airfoils presented in Table 4 allow us to make up some preliminary conclusions about preferable regions of airspeed values for each subset of the airfoil collection.

Airfoil	Integrated relative *L/D* losses for micro-UAV wing, ΔK %		
	Weight function, Figure 3	Weight function, Figure 4	Weight function, Figure 5
Eppler 61	42.15	54.51	68.19
Eppler 193	42.85	13.17	16.55
FS 60100	9.41	6.61	18.81
Göttingen 495	37.6	42.74	49.62
G 532	63.89	74.96	78.50
N-81	68.73	82.54	80.93
NACA 4512	14.27	5.99	2,58
NACA 4212	13.33	11.96	11.28
NACA 19	12.97	12.74	18.14
NACA 20	42.81	52.65	56.24
NACA 15	8.09	10.04	28.39
NACA 17	74.84	76.34	74.82

Table 4: Computational experiment results for collection of alternative micro-UAV airfoils with regard to the integrated approach

We can use the integrated approach (see Eq. 7) with weight functions *p(x)* defined with Figures 3, 4, and 5 to choose more precisely the most reasonable wing airfoil for micro-UAV. Appropriate simulation results are presented in Table 4.

As we can see the NACA 15 airfoil is preferable for the region with dominance of low speed flights (see Figure 3). This airfoil has minimal integrated relative *L/D* loss which equals to $\Delta K = 8.09\,\%$. As regards to the dominance of middle speed and large speed flights the NACA 4512 airfoil is preferable. It has $\Delta K = 5.99\,\%$ and $\Delta K = 2.58\,\%$ integrated relative *L/D* losses, respectively (see Figures 4 and 5).

Airfoil	Maximal relative losses ΔK_1, %	Reasonable airfoil
Eppler 61	96.6	
Eppler 193	36.8	
FS 60100	54.0	
Göttingen 495	82.8	
G 532	96.6	
N-81	30.0	
NACA 4512	94.3	
NACA 4212	77.0	
NACA 19	19,5	NACA 19
NACA 20	82.8	
NACA 15	36.8	
NACA 17	97.7	

Table 5: Collection of alternative airfoils for micro-UAV

We can apply also the guaranteed approach (see Eq. 8), which does not require weight functions *p(x)*. An estimation of relative *L/D* losses in this case is carried out for the whole region of airspeed values from 8 m/sec to 28 m/sec. Only maximal values of the losses ΔK_1 are essential for each airfoil in the guaranteed case. Simulation results to obtain ΔK_1 maximal values are presented in Table 5.

As we can see from Table 5 the smallest maximal relative *L/D* loss equals to $\Delta K_1 = 19.5\,\%$. This value belongs to the NACA 19 airfoil which is the most reasonable choice according to the guaranteed approach. This airfoil is not the best design alternative according to the integrated approach, however its relative losses are rather small. The losses values make up 12.97 %, 12.74 %, and 18.14 % for the weight functions presented on Figures 3, 4, and 5 respectively.

6 CONCLUSION

The problem of most reasonable selection for airfoil was stated and solved in this paper to ensure UAV efficiency for some range of its flight regimes, for example for some range of the micro-UAV airspeed values. The selection is carried out from predetermined collection of airfoils with known aerodynamic characteristics. Appropriate methodology was suggested to solve this kind of design problems using multitask approach.

The multitask approach is based on a set-theoretic statement of design optimization problem which allows to take into account diversity and uncertainty of UAV flight regimes as well as a set of effectiveness criteria.

We can apply the integrated multitask approach to select the most reasonable alternative airfoil if we have information about realization rate function *p(x)* for all flight regimes. Otherwise we can use the guaranteed approach, which does not require information about *p(x)* function.

Simulation results show us that suggested multitask approach to select reasonable airfoil enables us to enhance UAV efficiency due to improvement of its aerodynamic perfection.

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