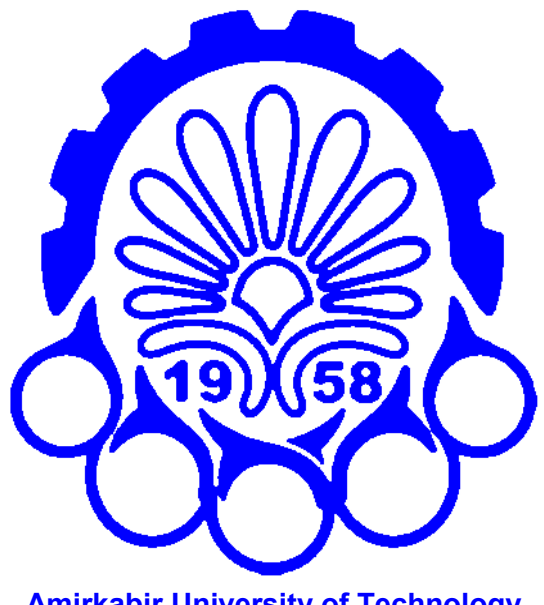




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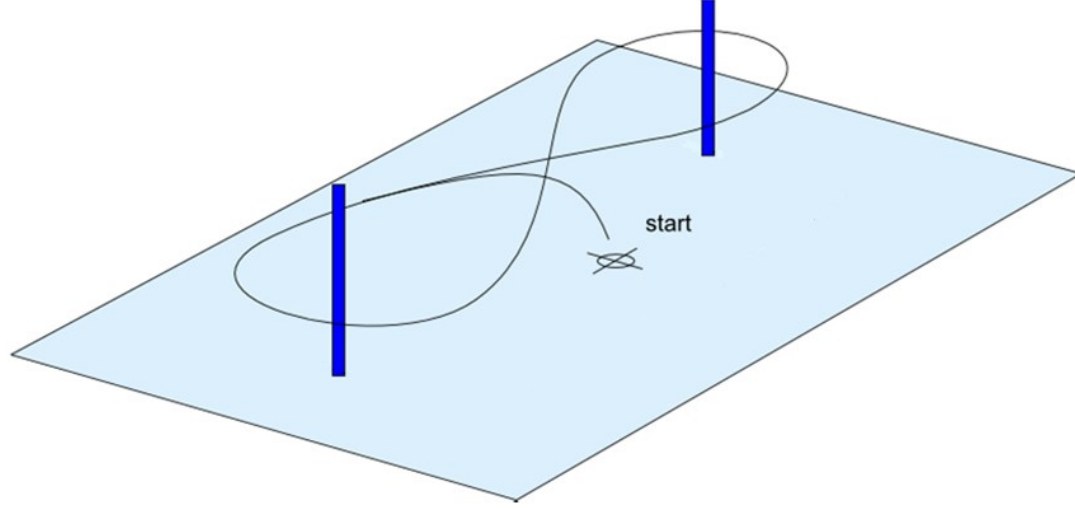
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Minimum Time Trajectory Optimization for Flying a Quadrotor in an 8-shaped Path

Abolghasem Naghash, Mehrdad Naghshineh, Ali Honari

1- Goal Definition

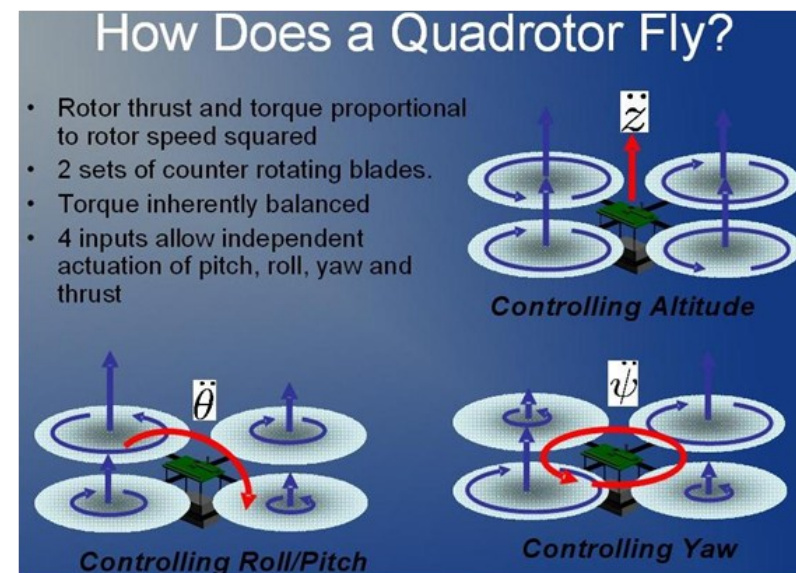
The goal in this paper is to solve the TOMP (time-optimal motion planning problem) for a maneuvering quadrotor for cruising around two focal points on a self crossed (8-shaped) path.



8-shaped path.

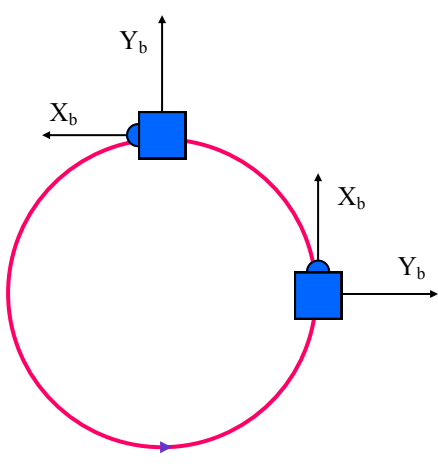
What path is time optimal?

2- Flight Principles



3- Flight Strategy

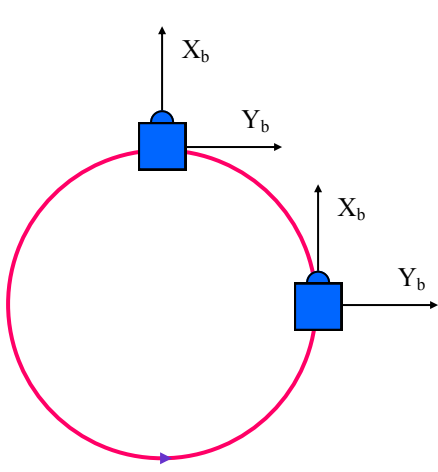
Two strategies to flight a quadrotor in an curved path:



Variable heading angle according to tangent direction of the path:

Advantage: To monitor videos for a pilot when MAV uses a front camera.

Disadvantage: Time response of the heading angle limits the speed.



Fixed heading angle:

Advantage for autonomous flying: It utilizes lower mobility functions.

Higher speed will be available.

In this work, the heading angle is assumed to be zero ($\psi=0$).

4- Point Mass 3DOF model

- Many quadrotors have internal control loops (PID) for flight stability.
- Guidance commands: Throttle (T), Roll (ϕ), Pitch (θ) and yaw (ψ).
- Modeling in cylindrical coordination system => Simplicity of constraints definition.
- 3DOF point mass model (assuming $\psi=0$):

$$\dot{q} = \begin{bmatrix} \dot{r} \\ \dot{\alpha} \\ \dot{z} \\ \ddot{r} \\ \ddot{\alpha} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} \dot{r} \\ \dot{\alpha} \\ \dot{z} \\ r\dot{\alpha}^2 + \frac{T}{m}(\cos\phi\sin\theta\cos\alpha - \sin\phi\sin\alpha) \\ -\frac{2\dot{r}\dot{\alpha}}{r} - \frac{T}{mr}(\cos\phi\sin\theta\sin\alpha + \sin\phi\cos\alpha) \\ -g + \frac{T}{m}\cos\phi\cos\theta \end{bmatrix} \quad (\text{Eq. 1})$$

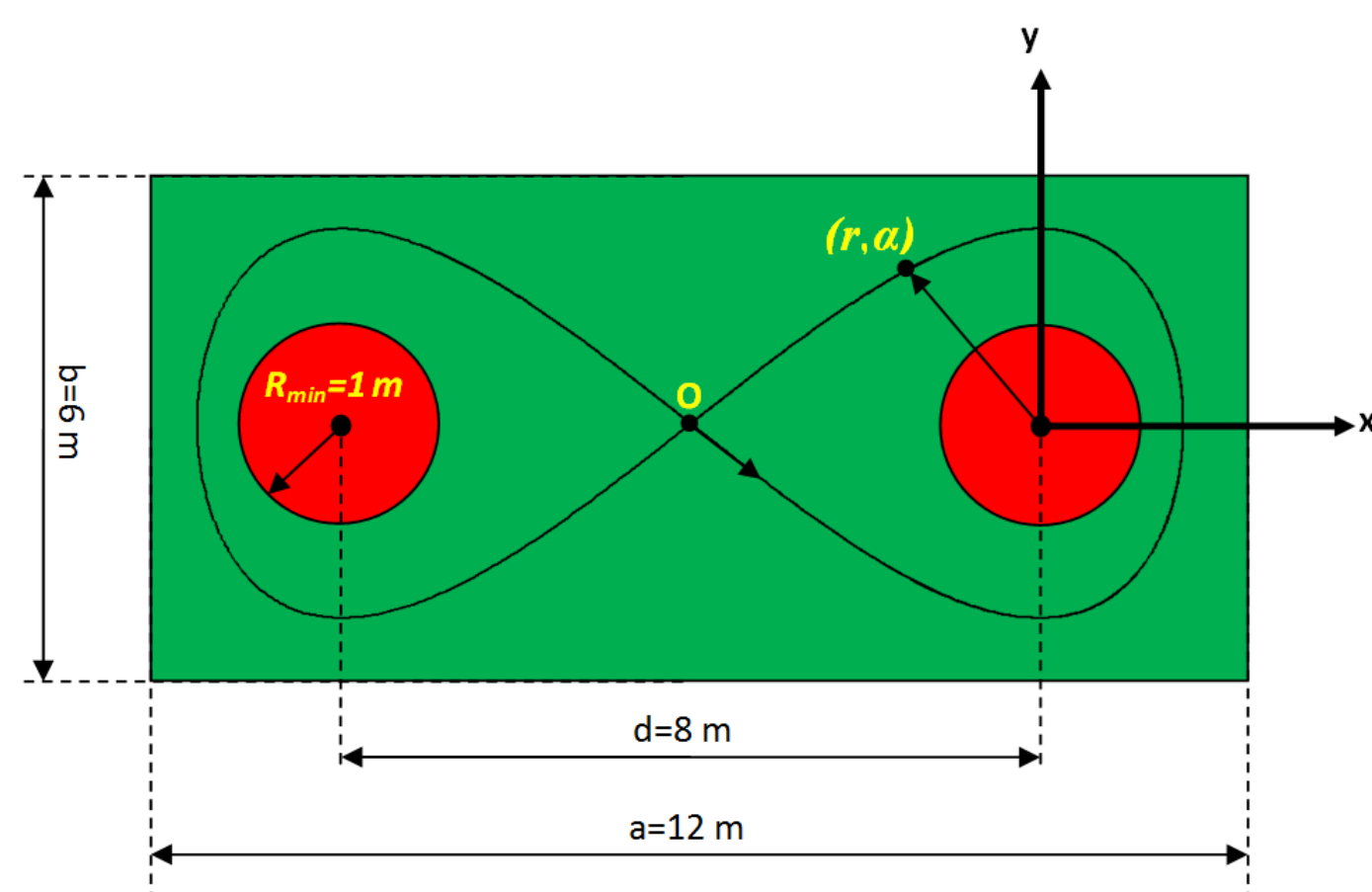
5- Control Constraints

- Control inputs: thrust force and roll and pitch angles (yaw=0).
- Constraints on control inputs:

$$\begin{cases} 0 \leq T \leq 2mg \\ -60 \leq \phi, \theta \leq 60 \text{ Deg} \end{cases} \quad (\text{Eq. 2})$$

6- State Constraints

- Path requirements => Mathematical definition of state constraints.
- Figure 2 presents a desired path similar to figure-8 in a horizontal plane.



Conditions and constraints of desired path.

Path requirements:

- Path should be within the selected area => Explicitly non-equality constraints.
- Path should be self crossed => This is a qualitative requirement.
- Path is symmetric => It can be braked into two closed loop half-paths around focal points.
- Self crossed (full path) translated to closed loop (half-path).
- The requirement of half-path to be closed loop can be addressed by defining suitable boundary conditions in a polar coordination system.

Tab.1: Summarize of constraints and boundary conditions on state variables.

Constraint qualitative definition	Mathematical representation
Flying within the rectangular area	$-\frac{d}{2} \leq r \cos \alpha \leq \frac{a-d}{2}$ $-\frac{b}{2} \leq r \sin \alpha \leq \frac{b}{2}$
Avoiding entering the prohibited circle	$r \geq R_{min}$
Fly in horizontal plane	$\ddot{z} = 0$
Closed loop around a focal point	$r(0) = \frac{d}{2}, \alpha(0) = -\pi$ $r(t_f) = \frac{d}{2}, \alpha(t_f) = +\pi$

How these boundary conditions make a closed loop half-path?

- The angle should change softly from $-\pi$ to $+\pi$, thus it is forced to cross a zero angle position.
- The MAV in the zero angle position must satisfy a minimum radius of 1 meter.

7- Summarize of Optimal Control Problem

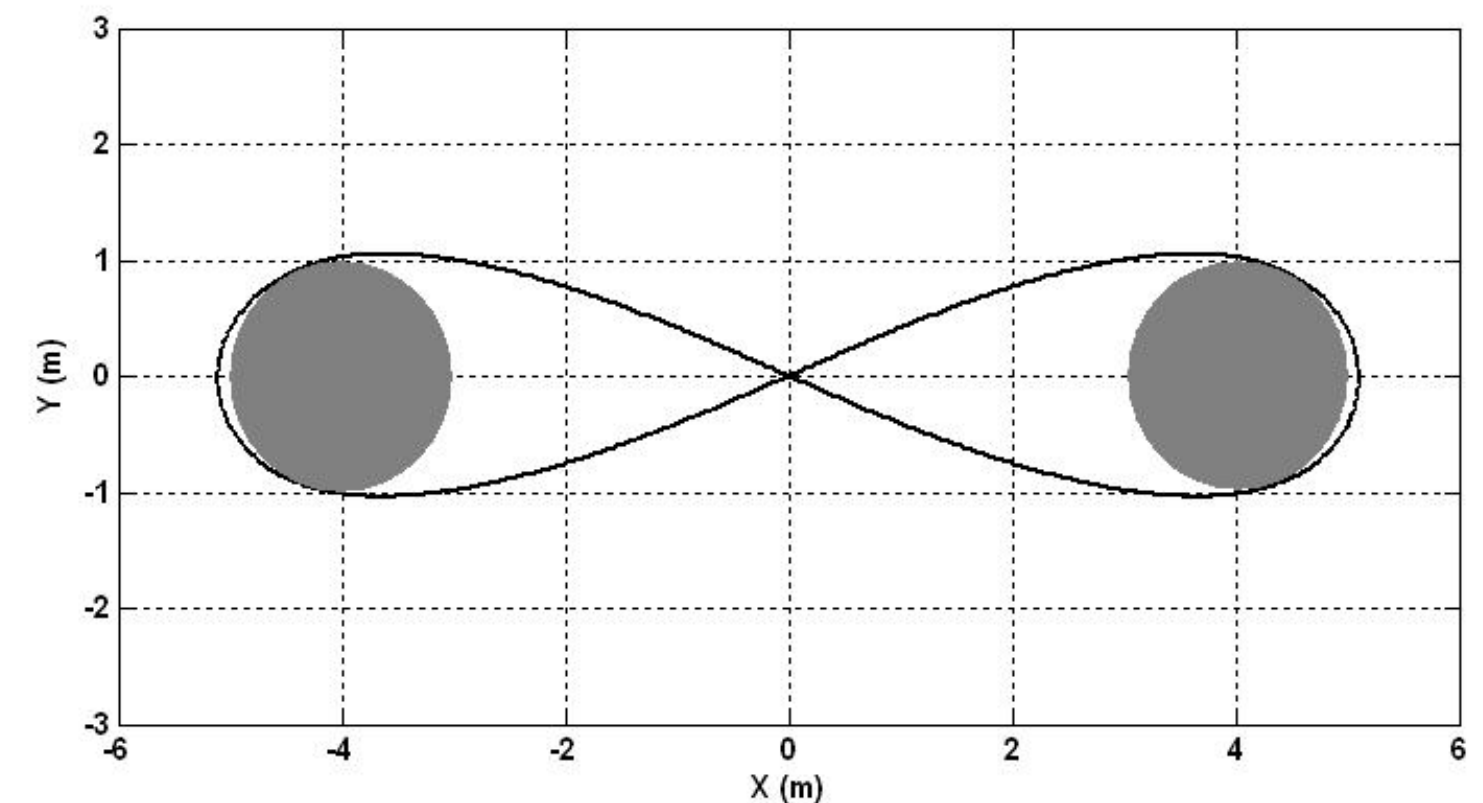
- Mathematical model: Equation 1
- Cost function: $J=t_f$ (final flight time)
- Control constraints: Equation 2
- State constraints: Table 1, rows 1,2,3
- Boundary conditions: Table 1, row 4

8- Solution Method

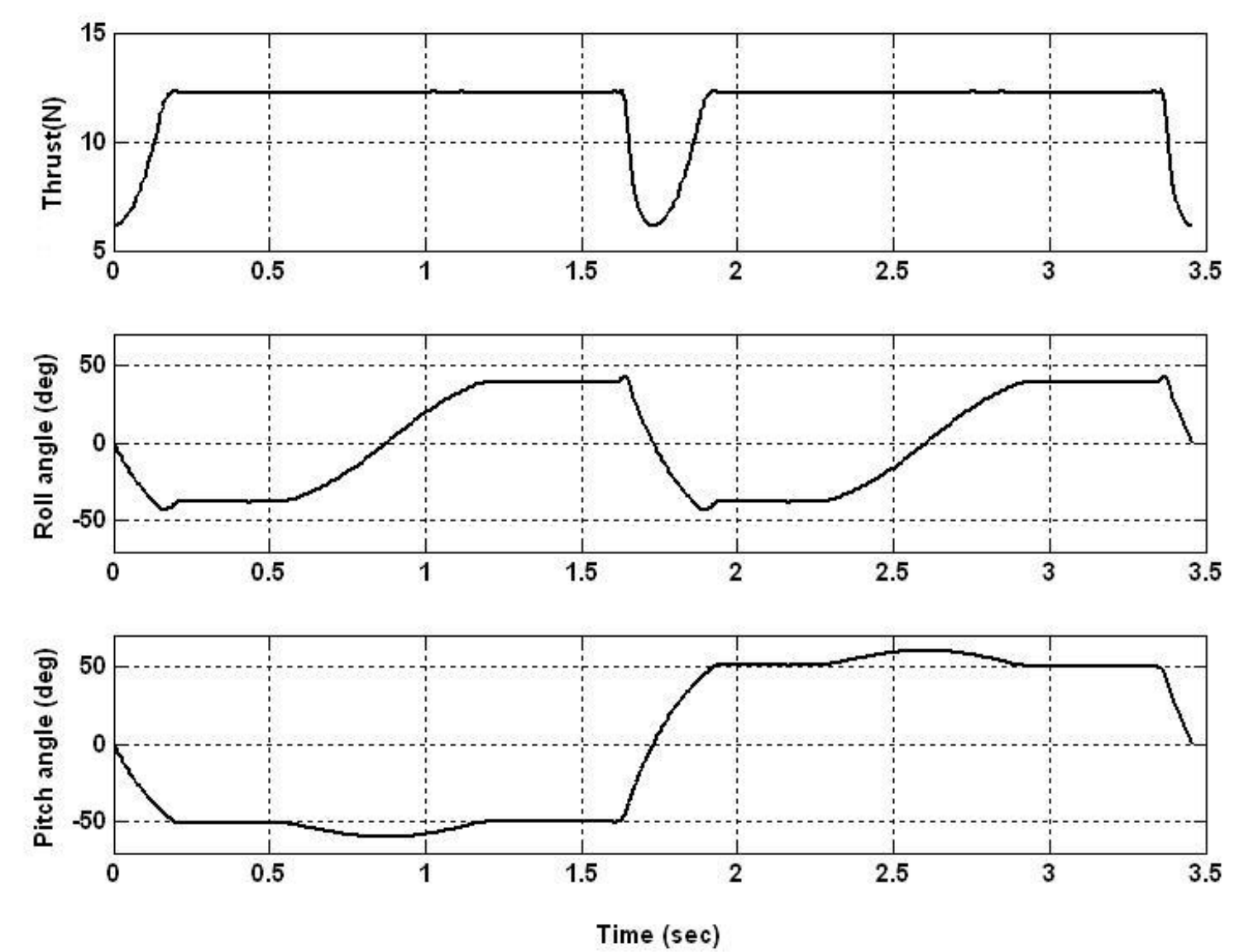
Optimal control problem is solved by using the GPOPS (General-Purpose Pseudospectral Optimal Control Software) which is an open source MATLAB code.

9- Numerical Results

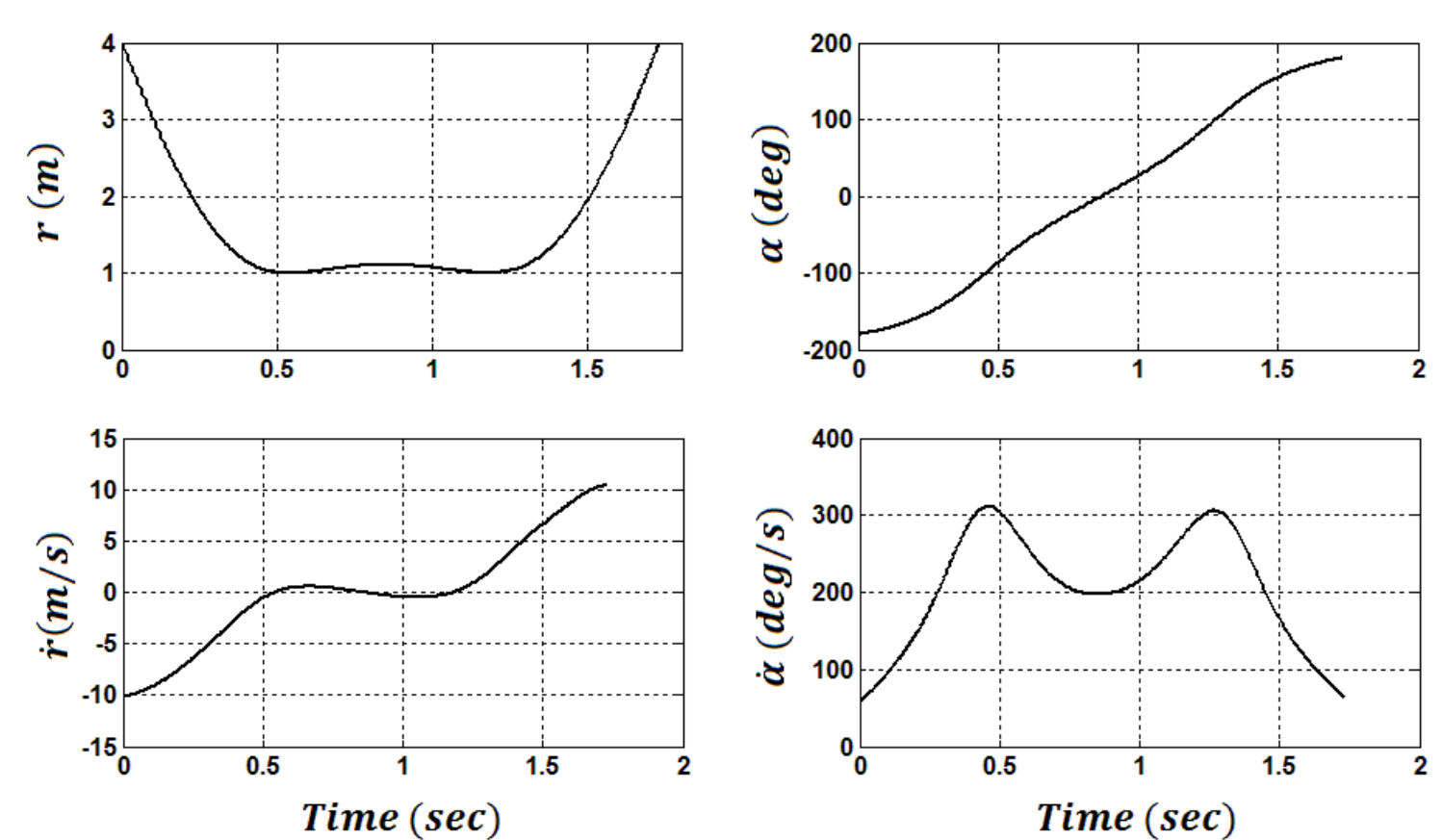
- A single 8-shaped path can be done in approximately 3.5 seconds.
- The optimal path passes as possible closest to the prohibited circles.
- Optimal states can be used for closed loop implementation.



Optimal path in x-y plane.



Optimal thrust force, pitch and roll angles vs. time.



Optimal states in a half-path