Implementation of copter propeller model to the problem of energy consumption minimization during lift phase

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ABSTRACT

The problem of minimum energy consumption for the lift phase of copter or VTOL airplane flight is investigated. Two models of propeller for the analytical investigation are proposed. Problem was solved analytically, the results analysis was made and propeller models accuracy was investigated.

1 INTRODUCTION

One of the tasks for copters and VTOL airplanes is to lift some payload to the defined altitude. This task can be a part of the flight mission or the main purpose of the flight. The best way of such a lift process is the flight with minimal energy consumption as it enables to use the accumulators with less mass and/or save energy for other parts of flight mission.

2 PROPELLER MODEL

Propellers used in copters are designed for long-time hovering. They have rather low Pitch/Diameter ratio (about 1:2 or less) that leads to the absence of flow separation on the blades at least during the hovering phase.

To describe the behavior of propellers, dimensionless characteristics are used [1]:
thrust coefficient

$$C_T = \frac{T}{\rho n^2 D^4}, \tag{1}$$

power coefficient

$$C_P = \frac{P}{\rho n^3 D^5}, \tag{2}$$

advanced ratio

$$J = \frac{V}{nD}, \tag{3}$$

propeller efficiency

$$\eta_{prop} = \frac{C_T J}{C_P}, \tag{4}$$

where T — propeller thrust, P — propeller power, ρ — air dencity, n — propeller frequency of rotation, D — propeller diameter, V — air velocity at infinity.

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The analysis of dimensionless characteristics of such a propellers shows that the thrust coefficient C_T decreases practically linearly with advanced ratio J up to zero value (it was shown in [2]) and power coefficient remains practically constant for zero and small values of J. To illustrate this, the plots for some propellers are presented in Figures 2–8 (see Appendix A). Figures 2– 6 show the caracteristics of different propellers with the same pitch and diameter. Figures 6 – 8 show the caracteristics of propellers of the same type, the same diameter and the same manufacturer with different pitch. More examples one can find in [1].

So, the following model is proposed: thrust coefficient decreases linearly with J, power coefficient is constant. In this case only three values are required for the propeller description:

- C_{P0} — power coefficient,
- C_{T0} — thrust coefficient at zero J,
- J_0 — value of J for zero thrust coefficient.

In this notation formulas for C_T and C_P can be expressed as

$$C_P = C_{P0} = \text{const}, \tag{5}$$

$$C_T = C_{T0}\left(1 - \frac{J}{J_0}\right). \tag{6}$$

According to (5), (6), if the copter is hovering or moving vertically with constant velocity V, power P and thrust of propeller T can be expressed as

$$P = kn^3, \tag{7}$$

$$T = an(n - bV), \tag{8}$$

where

$$k = C_{P0}\rho D^5, \tag{9}$$

$$a = C_{T0}\rho D^4, \tag{10}$$

$$b = (J_0 D)^{-1}. \tag{11}$$

From (8) one can find the rotational frequency of copter propeller as a function of T and V as

$$n = \frac{bV + \sqrt{(bV)^2 + \frac{4T}{a}}}{2}. \tag{12}$$

During the hovering or vertical lift with constant velocity the thrust must be equal to

$$T = \frac{mg}{N}, \tag{13}$$

where N is the number of propellers in copter, m is a copter mass, g — gravity acceleration.

3 ENERGY MINIMIZATION

Assume that copter must lift to the altitude h with constant velocity (excluding the beginning and the end of the lift), so lift time t is

$$t = \frac{h}{V}. \tag{14}$$

Total energy consumption is

$$E = \frac{NPt}{\eta}, \tag{15}$$

where η is the efficiency of electrical part of powerplant. Assume that η is constant. Substituting (12), (14) into (15),

$$E = \frac{kt}{8\eta}\left(\frac{bh}{t} + \sqrt{\left(\frac{bh}{t}\right)^2 + \frac{4T}{a}}\right)^3. \tag{16}$$

The condition of minimum of energy in this process is

$$\frac{dE}{dt} = 0. \tag{17}$$

Substituting of (13), (16) into (17) gives

$$t_m = bh\sqrt{\frac{2a}{T}} = bh\sqrt{\frac{2aN}{mg}}. \tag{18}$$

Condition (18) together with (12) gives the frequency of optimal lift n_m as

$$n_m = \sqrt{\frac{2T}{a}} = \sqrt{\frac{2mg}{aN}}. \tag{19}$$

It should be noted that this value of frequency is $\sqrt{2}$ times higher than the frequency for hovering.

Propeller power for optimal lift is

$$P_m = k\left(\frac{2T}{a}\right)^{3/2} \cdot N = \frac{k(2mg)^{3/2}}{\sqrt{N}}. \tag{20}$$

It is $\sqrt{8}$ times higher than the propeller power required for hovering.

Formulas (19), (20) give the method of defining the rotation frequency and power in climb. One can measure these values at hovering and simply multiply on the corresponding coefficients.

Also (19), (20) show that for the fixed diameter and geometry of propeller, the more propellers are used (in other words, the more the total area of all the propellers) the lower is the total power and the higher is the time of climb.

One more thing should be mentioned. Condition (18) correspondes to the maximal thrust at fixed V in expression (8).

The value of minimal energy E_m required for lift is

$$E_m = \frac{4kbThN}{a\eta} = \frac{4kbmgh}{a\eta}. \tag{21}$$

As mgh is usefull work in our case, $a\eta/(4kb)$ can be assumed as total effitiency of this process. Using (9)–(11) one can express (21) in the form of

$$E_m = \frac{4ThN}{\eta}\frac{C_{P0}}{C_{T0}J_0} = \frac{4mgh}{\eta}\frac{C_{P0}}{C_{T0}J_0}. \tag{22}$$

So the other definition of the process total eficiency η_{total} is

$$\eta_{total} = \eta\frac{C_{T0}J_0}{4C_{P0}}. \tag{23}$$

Analysis of (4)–(6) shows that

$$\frac{C_{T0}J_0}{4C_{P0}} \tag{24}$$

corresponds to the maximum of propeller efficiency within the model investigated, and corresponding value of J for this case is equal to $J_0/2$. From this one can make a conclusion that for minimum energy consumption during the lift the propeller must work at the regime of maximum efficiency.

It should be noted that the experimental data (see Figures 2–8) show that maximal propeller effitiency corresponds to $J = 0.63 \div 0.65J_0$. This is due to the fact that C_P begins to decrease at $J > 0.3J_0$. On the other hand, the rate of decrease is slow enough so the difference between our model and experimental data is not high, and one can use this model at least for the prelimenary analysys.

Another thing that must be discussed is that the minimal energy consumption at hovering fase is proportional to the so called figure of merit FOM, and [3]

$$FOM \sim \frac{C_{P0}}{(C_T)^{3/2}}. \tag{25}$$

For the fixed diameter the maximal efficiency increases with the pitch increase while FOM decreases with pitch increase. Figure 1 shows this dependence for the propellers APC SP 11×3, 11×4, 11×5 [1] (see also Figures 6–8 from Appendix A).

It is interesting to compare the climb velocity with the mean velocity of the air V_A moving through the propeller area during the hovering. From (10), (11), (14), (18)

$$V = \frac{1}{b}\sqrt{\frac{T}{2a}} = \sqrt{\frac{T}{2C_{T0}\rho D^2}}J_0.$$

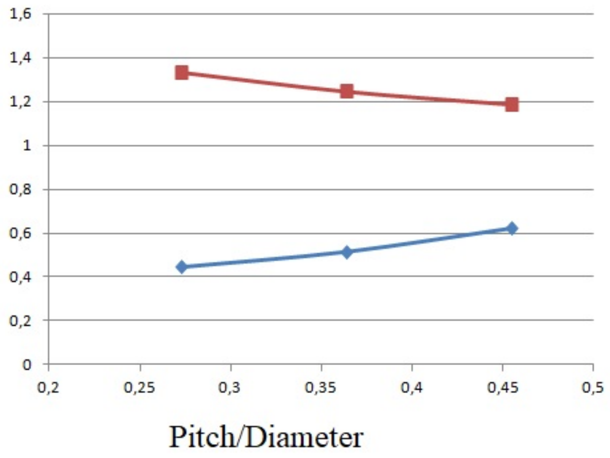

Figure 1: Efficiency (blue) and FOM (red) as functions of Pitch/Diameter.

As

$$T = \rho V_A^2 S,$$
$$S = \pi D^2/4,$$

so

$$V = V_A\sqrt{\frac{\pi}{8C_{T0}}}J_0 \simeq 0.63 \cdot V_A \frac{J_0}{\sqrt{C_{T0}}}.$$

For the propellers from Appendix A

$$V \simeq V_A.$$

This gives a simple method of optimal vertical velocity determination: measuring the air velocity past the propeller during the hovering gives the required result.

4 MORE PRECISE PROPELLER MODEL

In Chapter 3 it was noted that the propeller model with (5) gives not very precise results. So, the idea of this chapter is to propose and test more complicated but more precise model of C_P. From Figures 2–8 in Appendix A one can see that the value of C_P decreases with the increase of J and is equal to zero at some value of J_1. On the other hand, near the zero values of J the magnitude of C_P is practically constant. From this, the idea is to approximate the dependence $C_P(J)$ by the parabola. The preliminary investigations show that the most useful form of this dependence looks like

$$C_P = C_{P0}\left(1 - \left(\frac{J}{J_0}\right)^2 (1-\delta)\right), \qquad (26)$$

where

$$\delta = 1 - \left(\frac{J_0}{J_1}\right)^2.$$

For the analysis of previous chapter $\delta = 1$.

Using the formula (26) in finding the minimum of (15) one can obtain the following:

$$n_m = \sqrt{\frac{T(\sqrt{\delta}+1)}{a\sqrt{\delta}}},$$

$$P_m = 2k\left(\frac{T}{a}\right)^{3/2}\sqrt{1+\frac{1}{\delta}},$$

$$t_m = bh\sqrt{\frac{a}{T}(\delta+\sqrt{\delta})},$$

$$E_m = 2\frac{Th}{\eta}\frac{C_{P0}}{J_0 C_{T0}}(1+\sqrt{\delta}),$$

$$\eta_{total} = \eta\frac{J_0 C_{T0}}{2C_{P0}(1+\sqrt{\delta})}.$$

The value of advanced ratio for maximal efficiency is

$$J_m = \frac{J_0}{1+\sqrt{\delta}}$$

For $\delta = 1$ these formulas become as in Chapter 3.

The value of

$$\frac{J_0 C_{T0}}{2C_{P0}(1+\sqrt{\delta})}$$

corresponds to the maximum of propeller efficiency in the model investigated. Substituting values of δ from data of Appendix A gives the corresponding value of J as $J = (0.63 \div 0.65)J_0$. These values of J and corresponding values of maximal propeller efficiencies coincide with those from graphs.

As an example let's consider the APC 11X4 propeller. According to data from [1], the coefficients for this propeller for $RPM = 6000$ are: $C_{T0} = 0.95$, $C_{P0} = 0.34$, $J_0 = 0.57$, $J_1 = 0.68$. For this data $\delta = 0.297$, $\delta^{0.5} = 0.545$, $J_m = 0.368$ $(0.64J_0)$, maximal propeller efficiency is 0.515. Data from [1] give maximal efficiency of 0.517 at $J_m = 0.369$. The dependencies of C_T, C_P and efficiency are given by red in Figure 6 with the experimental data. One can see the good coincidence between the analytical formulas and experimental data. The accuracy and deviations of analytical formulas results are mainly due to the accuracy and deviations of experimental data that are used for the determination of the coefficients.

It is also useful to analyze the case when J_1 tends to J_0. For this case δ tends to zero, and, from this, t_m tends to zero while P_m tends to infinity but their product Em tends to its minimal value (as a function of δ), rotational frequency n_m and climb velocity tend to infinity while the total efficiency tends to its maximal value with respect to δ, J_m tends to J_0. The condition $J_1 = J_2$ physically means that there is no drag on the blades of propeller at zero lift. In other words, this condition corresponds to the "best", "ideal" propeller, and the best lift scenario for this propeller is the instantaneous lift with infinite power with infinite vertical velocity. Of cause, the drag of copter's construction is not taken into account here.

Now compare the formulas with those from Chapter 3. The optimal frequency of rotation is higher than in Chapter 3 while power and time of lift are lower. More important is that the efficiency is higher than in Chapter 3. This means that the formula (23) underestimate the real value of propeller efficiency. On the other hand, for the real values of δ that are only slightly less than unity the difference in formulas is rather small.

5 MAXIMAL CLIMB ALTITUDE

For the simplification of further investigation let's rewrite the formula for the minimal energy (21) as

$$E_m = \frac{mgh}{\eta_{total}}. \qquad (27)$$

The mass of accumulator m_{ac} is practically proportional to its maximal energy stored E, and proportionality coefficient is α, so

$$m_{ac} = \alpha E.$$

Assume that we can change the mass of accumulator to maximize the maximal climb altitude of the copter. If the mass of copter without accumulator mass required for climb is m_0 (m_0 can include the mass of other accumulators, required for other parts of the mission), then

$$m = m_0 + E_m. \qquad (28)$$

From (27) and (28)

$$E_m = \frac{m_0 gh}{\eta_{total}} \left(1 - \frac{\alpha gh}{\eta_{total}}\right)^{-1}.$$

The maximal climb altitude corresponds to the condition of

$$h = \frac{\eta_{total}}{\alpha g}.$$

For typical LiPol accumulator the stored energy is about 200 Watt·hour/kg. For the total efficiency $\eta_{total} = 0.5$ it gives the maximal altitude of 36 km. Up to now there is no aircraft flying at such altitude. Of cause, this altitude corresponds to very heavy copter and can't be realized and we haven't take into account the aerodynamical drag of construction, the air density and temperature change with altitude, the change of construction mass with the change of total mass and many other peculiarities (that can be analyzed in the future work), but this result shows that now copters with the moderate amount of accumulators onboard can reach rather high altitudes.

6 CONCLUSION

1. On the basis of experimental data two models of propeller for the analytical investigation of minimum energy lift mode for the copters and VTOL airplanes are proposed and investigated.

2. The task of minimum energy consumption lift mode for copters and VTOL airplanes is solved. Optimal propeller parameters and regimes are obtained. It is shown that for the energy consumption minimization the propeller must work at the regime of maximal efficiency and the form of propeller must be optimized for the maximization of efficiency.

3. It is emphasized that for the minimal energy consumption during the hovering phase the propeller must work and its forms must be optimized for the maximization of Figure Of Merit value and increasing the propeller efficiency decreases its FOM.

4. The maximal available climb altitude is evaluated. It is shown that it is higher than the typical altitudes of aircraft cruise flight, so practically all the required altitudes can be reached by copter.

REFERENCES

[1] https://m-selig.ae.illinois.edu/props/propDB.html

[2] *Serokhvostov S.V.* and *Churkina T.* One useful propeller mathematical model for MAV. Proc. of International Micro Air Vehicle conference and competitions 2011 (IMAV 2011), 't Harde, The Netherlands, September 12-15, 2011. Delft University of Technology and Thales, 2011

[3] *Robert W. Deters* and *Michael S. Selig* Static Testing of Micro Propellers. Materials of 26th AIAA Applied Aerodynamics Conference 18–21 August 2008, Honolulu, Hawaii. AIAA 2008–6246.

APPENDIX A: PROPELLERS DATA

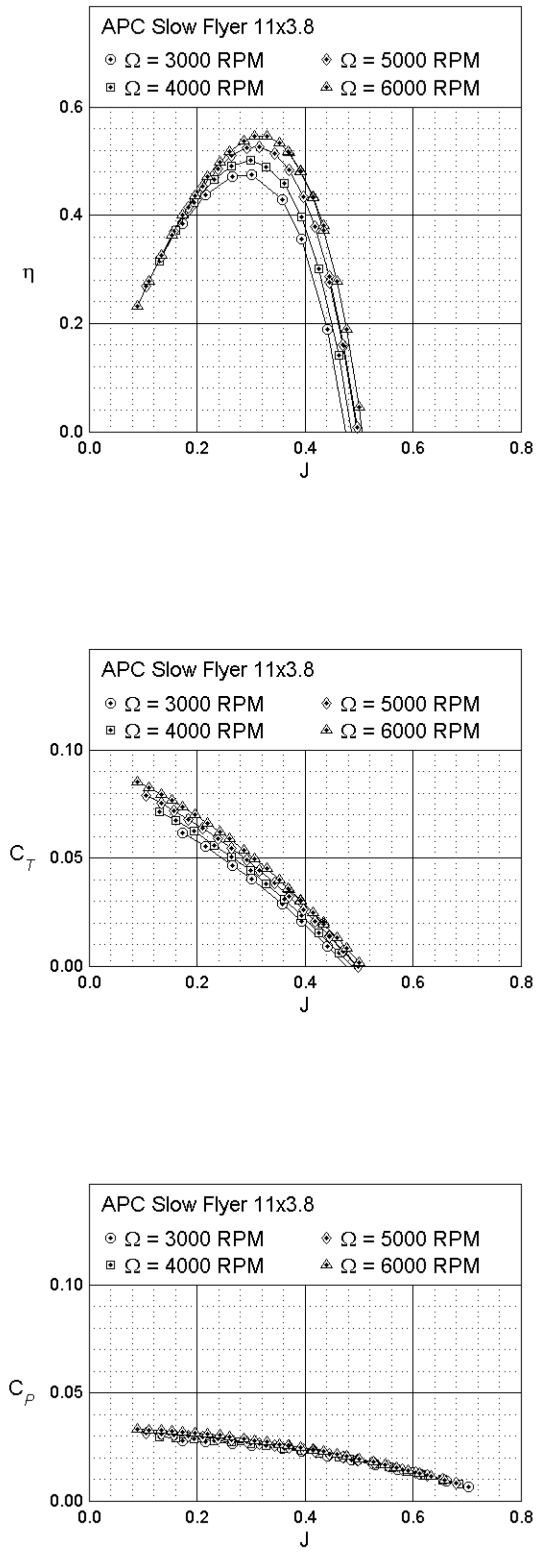

Figure 2: APC Slow Flyer 11x3.8 propeller characteristics. [1]

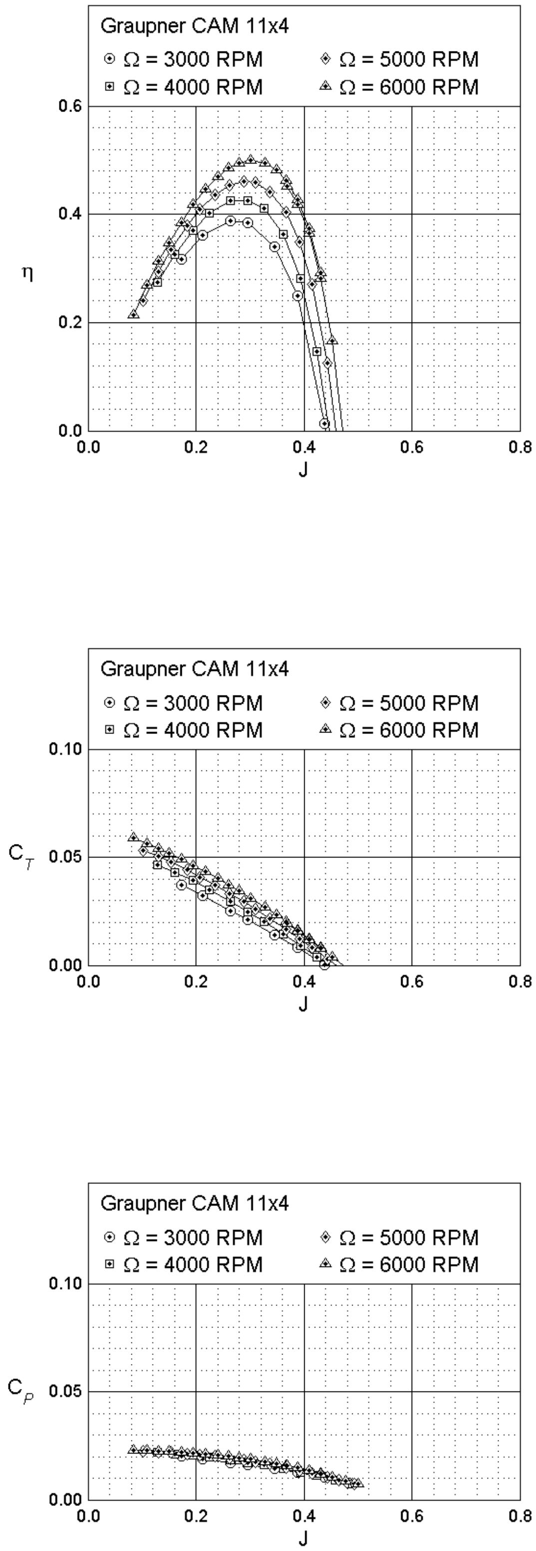

Figure 3: Graupner CAM 11x4 propeller characteristics. [1]

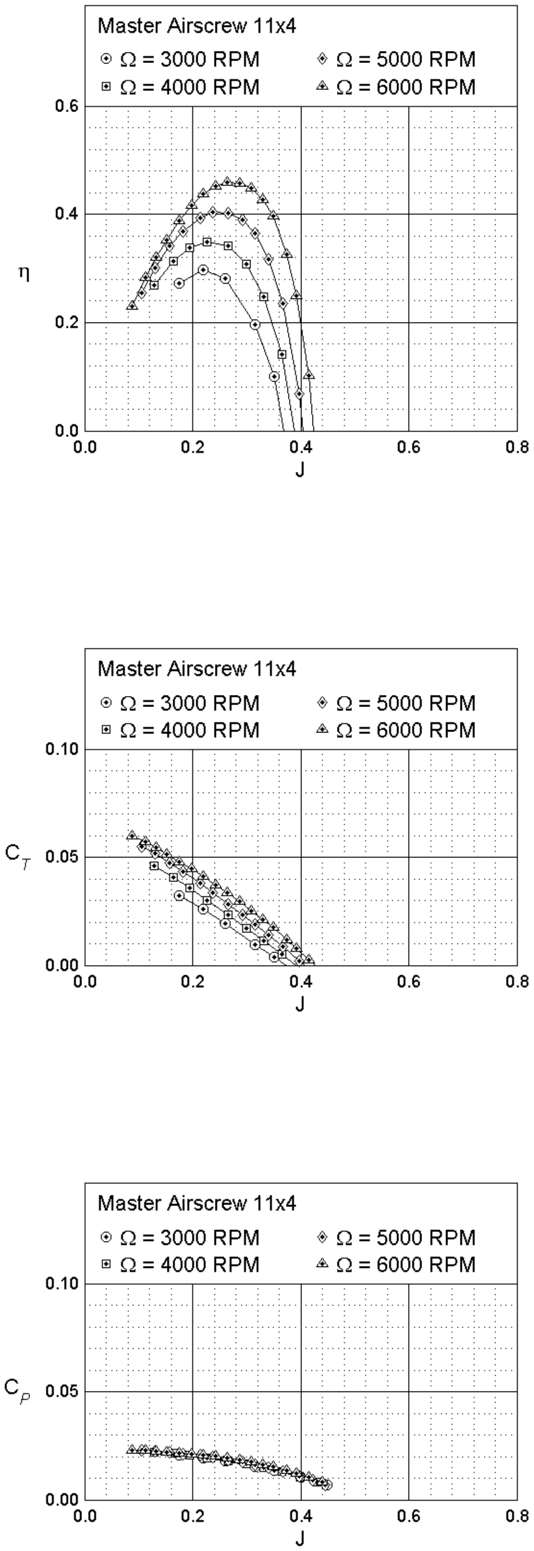

Figure 4: Master Airscrew 11x4 propeller characteristics. [1]

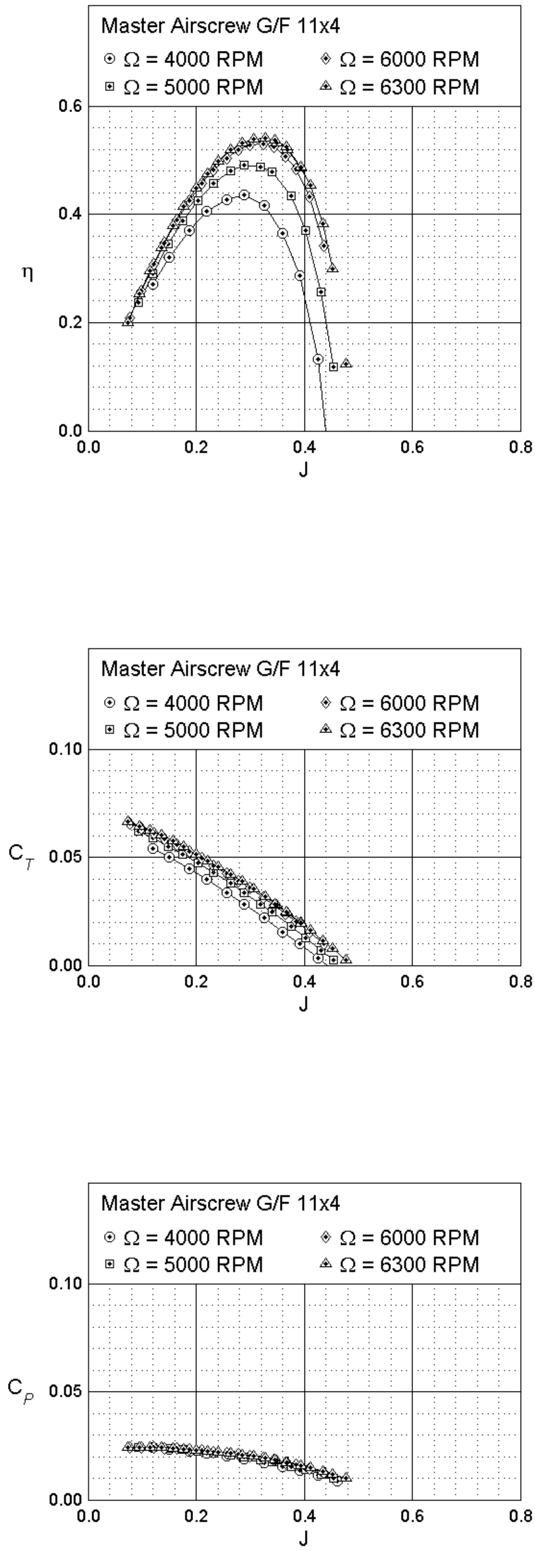

Figure 5: Master Airscrew G/F 11x4 propeller characteristics. [1]

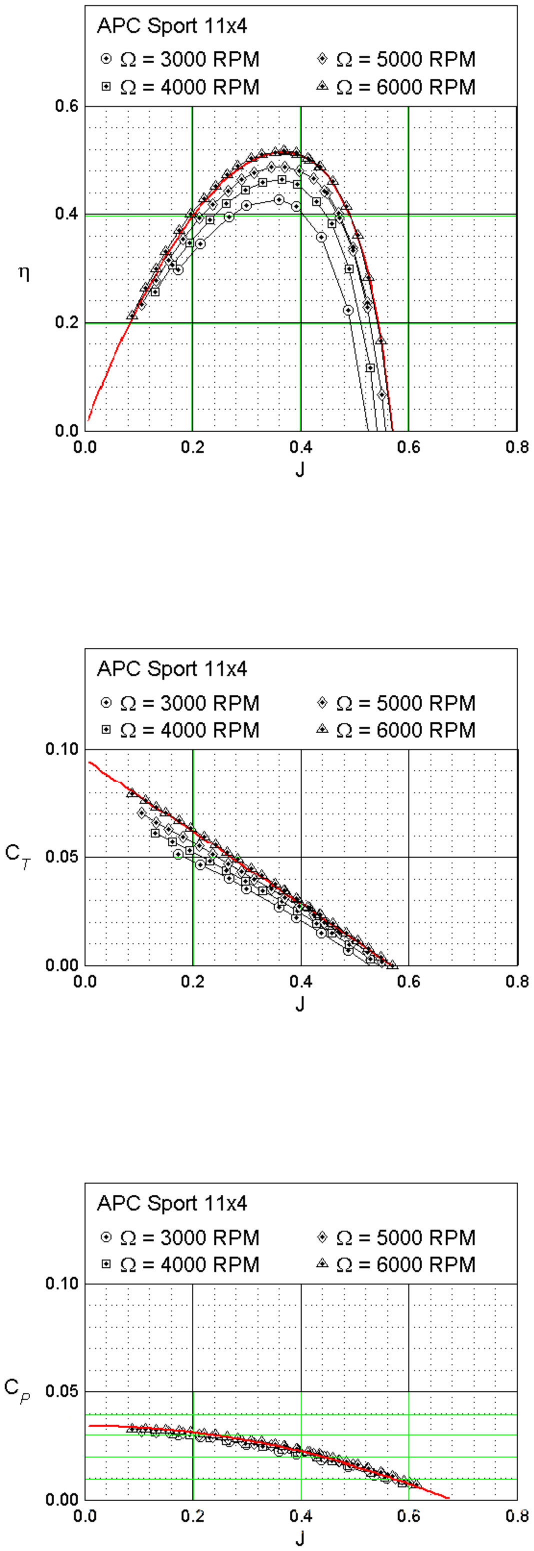

Figure 6: APC Sport 11x4 propeller characteristics [1]. Red lines are analytical formulas

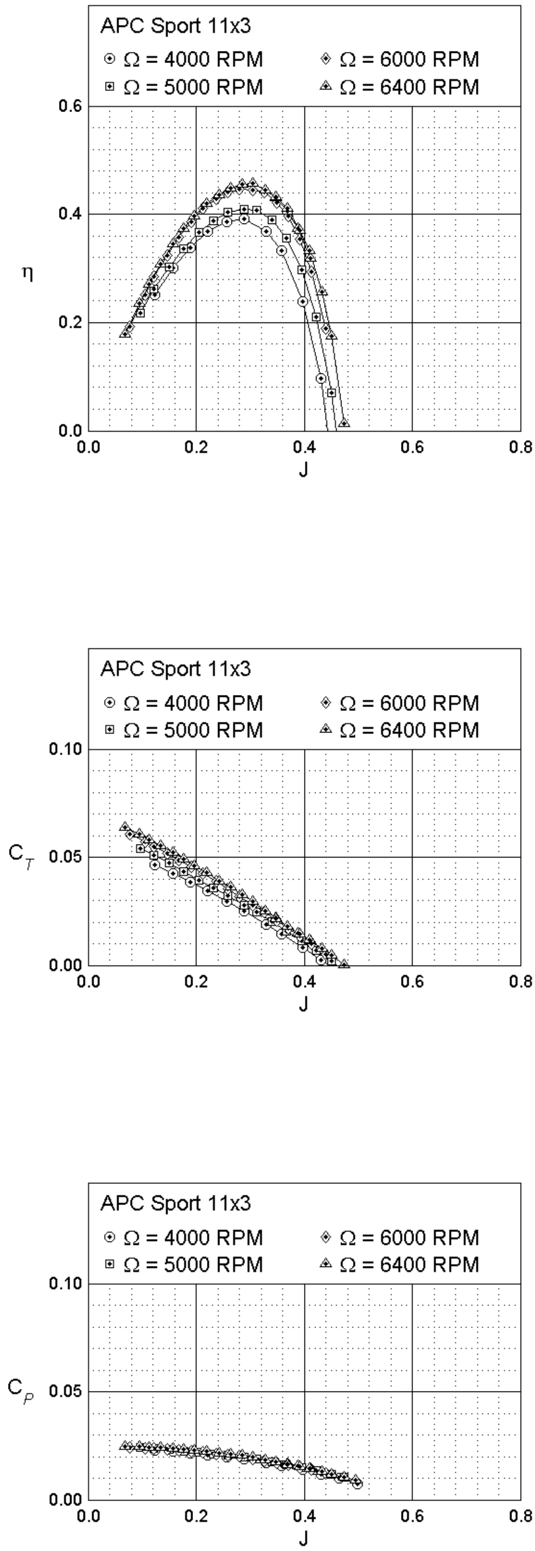

Figure 7: APC Sport 11x3 propeller characteristics. [1]

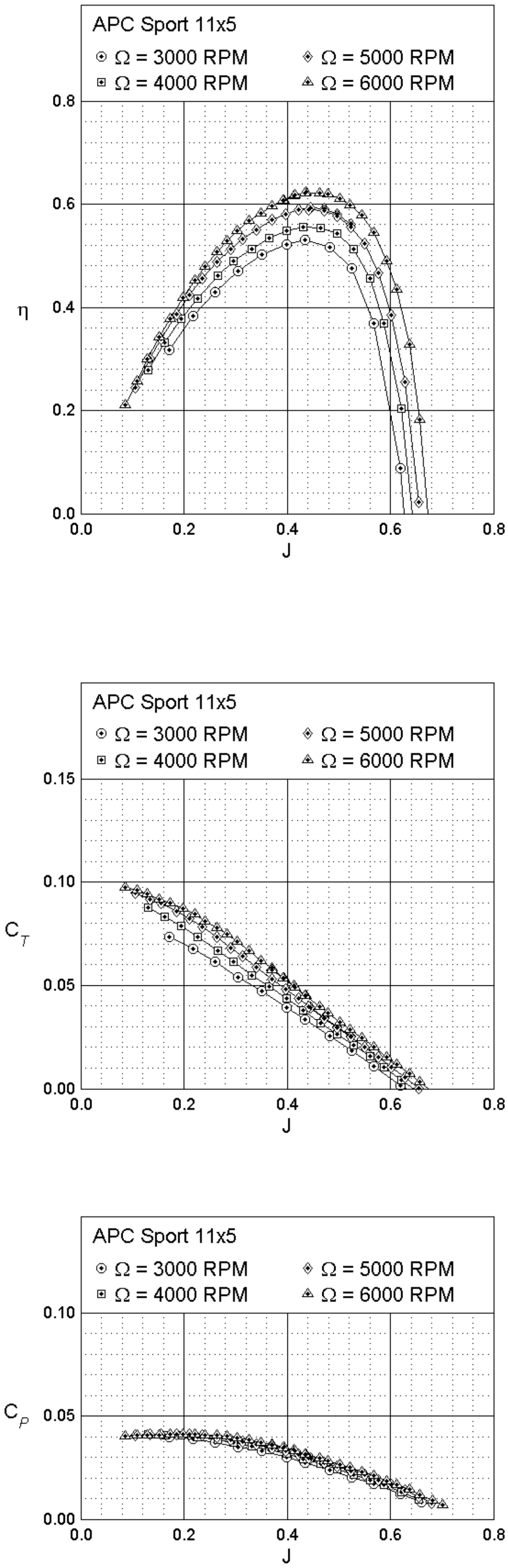

Figure 8: APC Sport 11x5 propeller characteristics. [1]