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Copter Size Minimization for the IMAV-2017 Competition in Record Breaking Session

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ABSTRACT

Optimization of aircraft design for the IMAV-2017 competition in Record Breaking session is investigated. Analytical research is conducted to understand the ways of optimization. A set of experimental designs was made and investigated to check the analytical results and to test technical solutions.

1 INTRODUCTION

According to the rules of IMAV-2017 competition in Record Breaking session, the goal is to lift the 0.5 kg payload for 1 minute at more than 50 cm above the ground. The winner is the aircraft with the lowest maximal dimension. Mass of the aircraft with the payload should not exceed 2 kg.

So, the optimization problem is to minimize the maximal dimension under some restrictions.

As a concept design the multi-copter was chosen. It was postulated that the maximal dimensions are defined through the number of propellers and propellers diameter. Some ways of optimization are

- to optimize the number of propellers
- to optimize the propellers number of blades
- to optimize the propeller blade shape and twist
- to optimize the diameter of propeller
- to optimize the motor
- …

2 ANALYTICAL INVESTIGATION

To understand the influence of the abovementioned factors the mathematical model of copter was made.

The characteristic cases for the investigation were chosen as helicopter, three-copter, quad-copter and hexa-copter. If the propeller diameter is D, the maximal dimensions are

- D for 1 propeller (helicopter)
- $2D$ for 3-copter
- $(1+2^{0.5})D$ for 4-copter
- $3D$ for 6-copter

For the following analysis we can present this dependence as function $f(N)$. The thrust T and power P of the propeller can be expressed as

$$T = C_T \rho n^2 D^4 \tag{1}$$

$$P = C_P \rho n^3 D^5 \tag{2}$$

where C_T — thrust coefficient, C_P — power coefficient, n — rotational frequency.

If the total mass of aircraft with the payload is m and the number of propellers is N, then, neglecting the interference between the propellers

$$P = \frac{C_P}{C_T^{1.5}} \frac{\left(\frac{mg}{N}\right)^{1.5}}{\sqrt{\rho}} \frac{1}{D} \tag{3}$$

In this case, the total power of N propellers is

$$P_{sum} = \frac{C_P}{C_T^{1.5}} \frac{(mg)^{1.5}}{\sqrt{N\rho}} \frac{1}{D} \tag{4}$$

so

$$D = \frac{C_P}{C_T^{1.5}} \frac{(mg)^{1.5}}{\sqrt{N\rho}} \frac{1}{P_{sum}} \tag{5}$$

Maximal dimension MD will be

$$MD = f(N)D = \frac{f(N)}{\sqrt{N}} \frac{C_P}{C_T^{1.5}} \frac{(mg)^{1.5}}{\sqrt{\rho}} \frac{1}{P_{sum}} \tag{6}$$

If the electrical efficiency of powerplant (motor, controller, accumulator etc.) is η, then the total energy E in the accumulator required for the flight during the time t is

$$E = \frac{P_{sum} t}{\eta} \tag{7}$$

The mass of accumulator is practically proportional to the energy stored (and also depends on the maximal current of accumulator). From this, we can accept that the accumulator mass is proportional to the total power.

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In the first approximation we can also assume that for all the other parts of powerplant (motors, controllers etc.) the mass is proportional to the power. In this case we can say that

$$m = m_0 + \alpha P_{sum} \tag{8}$$

where α is some coefficient of proportionality, m_0 is the part of total mass that does not change with P_{max} change. Finally

$$MD = \frac{g^{1.5}}{\sqrt{\rho}} \frac{f(N)}{\sqrt{N}} \frac{C_P}{C_T^{1.5}} \frac{(m_0 + \alpha P_{sum})^{1.5}}{\alpha P_{sum}} \alpha \tag{9}$$

So, roughly, the function of MD is the multiplication of function depending on N, function depending on the propeller geometry and function depending on the powerplant characteristics. From this, one can analyze and minimize these functions separately.

First of all, for the number of propellers, the corresponding function $f(N)/N^{0.5}$ is equal to

- 1 for helicopter,
- 1.154 for 3-copter,
- 1.207 for 4-copter,
- 1.225 for 6-copter.

The best design is helicopter (1 propeller) but it was not taken into account the questions of helicopter stability. Some additional "devices" must be implemented that can increase the dimensions or mass of aircraft, so helicopter in reality is not too good. The difference between 3-copter and 4-copter is about 4.6%. But 3-copter now seems more complicated, so one of the conclusions is that the quad-copter can be good compromise for this task.

The value of $C_P/C_T^{1.5}$ (inverse figure of merit) depends on the geometry of blades and the number of blades NN. In first approximation one can assume that both C_P and C_T depend linearly on the number of blades (in reality the dependence for C_T is lower than linear and for C_P is higher than linear). In this case the value of $C_P/C_T^{1.5}$ changes as $NN^{-0.5}$. From this point of view, the more blades the better. Practically the same effect is due to the increase of blade width. But for some number of blades or some width due to the effects not taken into account there must be minimum $C_P/C_T^{1.5}$. Also, from geometry, for some number of blades and some width the blades will touch each other, and this will be geometrical limitation on the number of blades.

So, the another conclusion is to use multi-blade propellers. Also one of promising solutions is to cut the blades of larger diameter propellers.

As for the propeller blade twist, for the analysis one can use the results of [1]. For the set of small propellers one can see that it is possible to choose the propeller with the high figure of merit for different diameters.

For the third function in dependency of MD the minimum with respect to P_{max} corresponds to the condition of

$$\alpha P_{max} = 2m_0 \tag{10}$$

Corresponding value of function in this case is $3^{1.5}\alpha m_0^{0.5}/2 = 2.6\alpha m_0^{0.5}$. If we take

$$\alpha P_{max} = m_0 \tag{11}$$

then the function is $2^{1.5}\alpha m_0^{0.5}/2 = 2.83\alpha m_0^{0.5}$ The difference is about 9%. So, the optimal mass of copter for this competition can be estimated as 1–1.5 kg. One can see that it is within the limitation of 2 kg.

3 TECHNICAL PROBLEMS

From (9) the maximal dimension depends on the value of α. This parameter includes such factors as powerplant efficiency, dependency of accumulator mass on the maximal current and some other factors. So, special attention must be paid on the question of choosing the motor characteristics and accumulator characteristics.

For example, for one series of motors one can choose the different Kv. On the one hand, the motors with higher Kv seems more powerful. On the other hand, for higher Kv the reaction time is longer. This make the stabilization less effective and requires some additional power. Also, it is known that for the fixed shaft power at fixed frequency the best efficiency corresponds to the motors for which the maximal power is several times higher that required. So, the motors with the best efficiency are heavier than less effective ones. This means that lighter motor can consume more power, and the accumulator for such motor must store higher amount of energy and thus be heavier. Other thing is that higher power for the same accumulator voltage requires higher current. It is well known that accumulator with higher maximal current (for the fixed voltage and capacity) has higher mass. One can see that some compromise must be found for this situation.

Another thing that must be taken into account is the restrictions on the motor maximal frequency. Decreasing the propeller diameter for the fixed thrust leads to the frequency increasing. So, the motors with the higher working frequencies for the fixed shaft power are required. On the other hand, such a motors can be not existing in the market.

Unfortunately, now it is practically impossible to describe analytically all these peculiarities, so the optimization must be made for the discrete set of motors and accumulators.

Up to here there were no words about the frame. We assume that in the first approximation the mass of airframe is also proportional to the total power.

Another factor not taken into account is the dimensions of accumulator. The sizes of accumulator of required capacity can be comparable with the dimension of propeller. In this case it is required to make some additional frame for it.

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It is evident that all these conclusions must be proved by the experiment. Some preliminary experimental investigations were made previously.

Also, some investigations are required for the designs with the propellers situating not in one plane. This enables to make lower dimension for the same propeller diameter due to the "intersection" of propellers.

Some peculiarities can occur due to the accumulator properties. First of all, for this task not only the capacity is important but also the maximal current. This is because of the fact that the accumulator must provide the required current that can be high enough. The second reason is that for the high values of current the power losses increase due to the accumulator's internal resistance. This leads to the lower accumulator efficiency. Also it is well known that the accumulator capacity depends on the discharge current. To diminish these factors one can use the accumulators with high maximal current but these accumulators have lower energy capacity.

The next problem for the accumulator is the dependence of voltage on the accumulator charge. It is well known that the fully charged LiPo accumulator has the voltage of 4.2 Volt and fully discharged accumulator has the voltage of about 3 Volt. This means that the maximal power is diminishing during the flight. So, it can be the situation that the aircraft that can stay in flight at the beginning of flight drop down after some time with some amount of energy in the accumulators.

Another factor is that copter must have some extra voltage in accumulator (comparing to the flight in perfect conditions) for the stabilization and manoeuvres.

All these reasons lead to the fact that the "real" construction will be not so optimal as the theoretical one.

4 COPTER DESIGN AND EXPERIMENTS

So, our goal was to create an aerial vehicle (AV) capable of lifting 0.5 kg cargo load to a height of at least 1 meter while staying in the air for more than 1 minute. AV geometric dimensions (maximum horizontal distance between its elements including propellers) should be minimized to an extent possible. In the above chapter we have provided the mathematical calculations associated with defining the optimal number of propellers and their blades, as well as the composition of other AV elements. We have found that the less number of propellers leads to the vehicle's smaller dimensions at the same level of thrust (subject to identical propeller characteristics for each design), so we decided to use a quadcopter as one of the most universal and sustainable designs for unmanned aerial vehicles.

At the next step we have selected powerplant and other electronic components. For this purpose we had to preliminary estimate the weight of AV, as follows:

- We estimated the battery capacity for similar purposes and this resulted with max. value 1300 mAh and 16 V (4S) (the weight of this battery type is about 170 g);
- The weight of AV carbon fiber frame was estimated to max. 80 g;
- It was decided to use a ready-made solution by installing integrated electronic speed controllers (ESC) and flight controller weighting 22 g;
- Receiver RC 10 g;
- Mounting equipment 40 g.

Taking into account 500 g of additional cargo, the total AV weight is estimated at 822 g, without motors and propellers which selection is discussed in more detail below.

One of the main methods to reduce an AV size is to use a propeller with minimal diameter taking into account allowable drop of thrust and related factors. Therefore we were selecting motors and propellers in parallel. It was decided to make the investigation "step by step", from simple case to more complex to test the solutions one by one. This gives more clear understanding of each specific factor.

We started with type BrotherHobby Tornado T1 1407 3600KV motors and 4045*3 propellers (hereinafter the first figure of a propeller model indicates the diameter in inches and the two last figures indicate its pitch). At 15 A current this type of motor with this propeller is capable of producing 535 g of thrust (data are provided by the motors manufacturers), which corresponds to total thrust of 2 kg of all 4 motors. The weight of AV equipped with these motors is 906 g. The required 60 A current is provided by the selected battery capable of generating a current up to 120 A. The estimated thrust of four motors without evaluation of the interference between the motors themselves, cargo and other parts of AV, exceeds the AV weight more than two times.

The experiment was carried out.

A box suspended under the AV (Figure 1) was selected as the cargo (the box larger face was positioned horizontally). AV took off at 90% of motors load, which reached 95% in stable flight, after 35 s of flight one motor wiring could not stand the power supply and AV fell down because of the motor burn-out.

When the cargo position was changed (the box smaller face was positioned along the air flow from propellers), AV could hardly manage to stay in the air for 1 minute required. (Figure 2). These experiments have helped us to find out that one of the main factors effecting AV performance with cargo is the cargo position (cargo's the smallest face should be positioned perpendicular to the air flow produced by propellers to reduce the aerodynamic force on cargo surface). Besides we considered 1407 3600 KV motors to be not powerful enough for the purpose. So, one of the conclusions is that the size and shape of cargo will have a major impact on AV power plant performance, so the AV frame should be designed for a specific cargo. Next, we considered two options for resolving this problem:

Figure 1: Copter of first design in flight with cargo fixed ”horizontally”

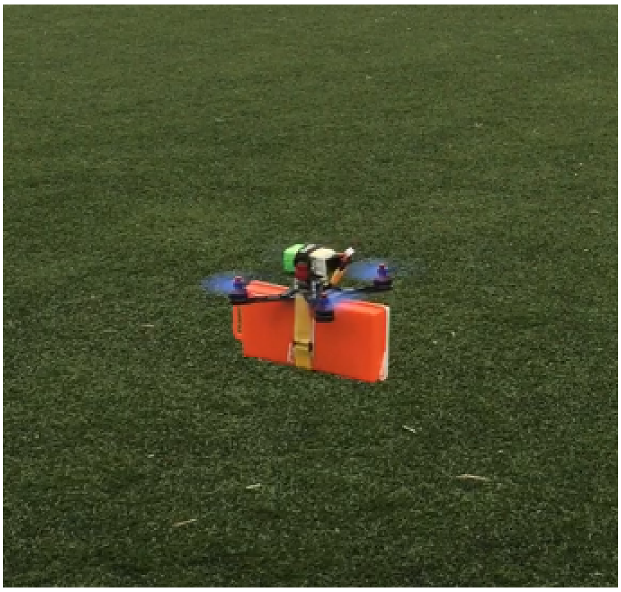

Figure 2: Copter of first design in flight with cargo fixed ”vertically”

Figure 3: Double deck frame

- Extension of the quadcopter main diagonal to divert the air flow produced by propellers from the cargo surface;
- Use of more powerful motors and, accordingly, larger propellers.

To select a more efficient option we have conducted the experiment as follows: by using motors of Titan TS2307-2300KV type with 5046*3 propellers (of higher diameter comparing to the first design) we reduced the frame diagonal keeping the AV size unchanged (as it was in the first experiment).

We carried out a similar flight with the same cargo fixed along the air flow. The AV robustly stayed in the air for 1 minute at 65% of motors load.

Thus, we have found that for this specific shape of cargo the most efficient solution is to use more powerful motors and bigger propellers, despite the increase of interference (overlapping) effect, and this finding is proved by the 1 minute stable flight of AV, which had the same size.

Having selected 2307 2300KV motors and 5046*3 propellers we considered the two following ways to make the AV size smaller:

- To reduce the propellers diameter by partial cutting the blades of existing ones;
- To minimize the frame size.

To minimize the frame size a double deck frame was designed (Figure 3). In this case the AV minimum size can theoretically be equal to $2D$, where D is the propeller diameter. In our case, when using 5045 propellers the AV size can be equal to 25.4 cm (10 inches), which is optimal for the design of double deck frame and these propellers.

At the time of these experiments the size and shape of cargo to be lifted by the AVs of the competition participants were published on the IMAV web-site. The width of the cargo

Figure 4: Copter with cargo on threads

main part is 180 cm, the length is 580 cm. As its weight is 500 g, the cargo is strongly subjected to the wind and thus heavily spoils the aerodynamics of AV + cargo system.

We have made the model of this cargo at the scale of 1:1 to investigate the influence of its shape and inertia.

When the cargo was fixed under the bottom of our AV, it was not possible to lift it because of the strong counter force from the propellers air flow (there was an appropriate experiment). We may say with confidence that our system of AV with the cargo of this size and shape can not operate efficiently, if the cargo is fixed under the AV bottom.

We considered the following ways to solve this problem:

- Suspension of the cargo on threads to reduce the force acting on it;
- Rigid fastening of cargo on a long support underneath AV;
- Rigid fastening of cargo above the AV, where the impact of air flow force produced by the propellers is much less.

When the cargo was fixed with a 30cm-long cable (see Figure 4) the most of the air flow still affected the cargo. Under these conditions the AV took off at 80% of motors load, and the entire system fluctuations appeared immediately making the AV control extremely difficult and unsafe. Then we have tried to extend the cable, assuming that it would result in reducing the airflow impact and the fluctuations of the system would become acceptable for the safe AV control. However even using a cable of 1.5 m length we did not achieve the desired result.

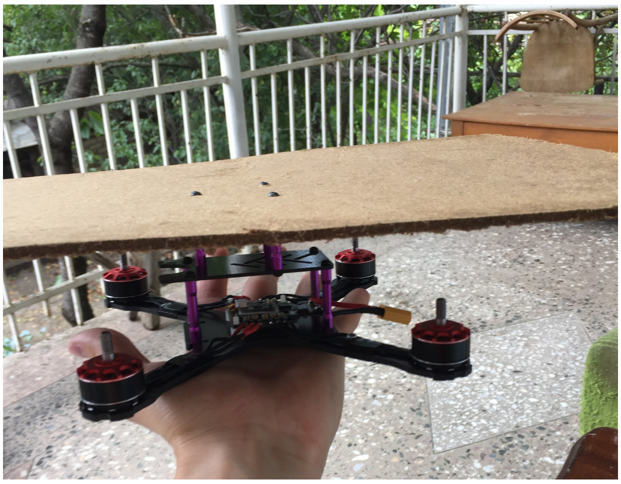

Figure 5: ”Cargo over AV” first design

The last option for fixing cargo is not applicable for our system, as there is no place on the AV to mount the fasteners for the cargo.

But to test the idea of ”cargo over AV” we assembled a quadcopter with a frame diagonal 210 cm long and fixed a board with dimensions declared for the cargo to be used in the competition. We did not overload the AV in the first experiment, therefore the weight of the board was 250 g instead of 0.5 kg (see Figure 5). In the first case the distance between the board and propellers plane was 5 cm. The AV took off at 65% of motors load. Then we increased the distance to 10 cm (Figure 6), and the take-off was operated at 50% of the motors load. It can be said with confidence, that when cargo, which size is similar to that of AV, is fixed over the AV, such operation is much more efficient than with cargo fixed beneath the AV. However ”cargo over AV” operation has its shortfalls too: the system centre of mass shifts upward thus affecting the stability of AV in the flight.

When we had completed the motors selection and the cargo loading and fixation scheme, we started designing the AV that meets our requirements.

The task of optimizing the AV size in this case becomes more complex and from our point of view may be divided into 2 parts:

1. Layout of all 4 motors at the same height level. Then the minimum size of AV that can be achieved is $2.4D$. This leaves a square form gap which can be used to install the battery below the propellers plane, which in turn contributes to lowering the centre of mass and increasing the AV flight stability.

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Figure 6: "Cargo over AV" second design

2. Layout of the motors at different levels (using double deck frame), then the minimum size of AV will be $2D$, as already mentioned above. However, the implementation of this scheme is somewhat more complicated. If the distance between motors reduced, AV flight stability becomes more difficult to be achieved due to decreasing of relative force moments. This may lead to the burn-out of motors, which will operate at 80-85% load level with heavy interference and the specified cargo. In this case it will also be impossible to lower the battery installation for the appropriate lowering AV centre of mass.

As the first step we have chosen the first type (Figure 7). Then, for the realization of this the new frame was designed. The drives were fixed in one plane, accumulator was placed below the propellers' plane. Also the special platform was made for the cargo fixing. Experiment was conducted with the cargo with the form and the dimensions required. The flight was stable and the motors' power was less than 60% of maximal value. As the results were very good it was decided to realize the second type. For this it was not necessary to make double-deck frame, we have used spacers to make two diagonal motors 4mm lower than others. This was enough to mount the motors as close as possible (Figure 8). In this case the final maximal AV dimension has become 26.8 sm (10.6 inches), see Figure 9. The test flight has been conducted, and the aircraft flight time was 1 min.

Some parameters of the final design are

Accumulator mass — 170g

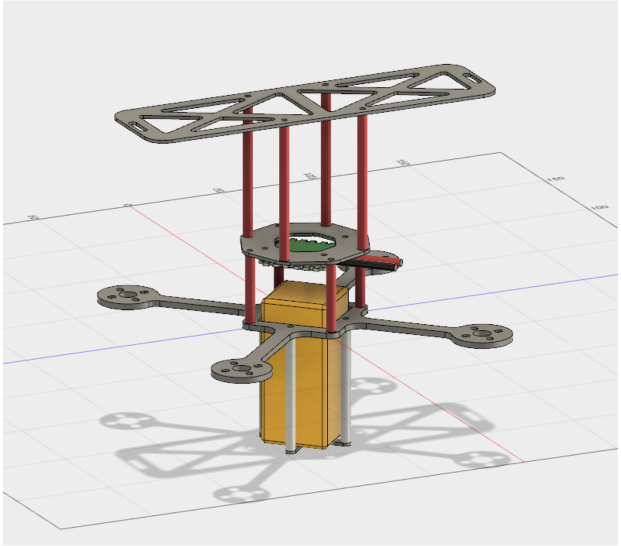

Figure 7: Airframe design for the cargo above the copter.

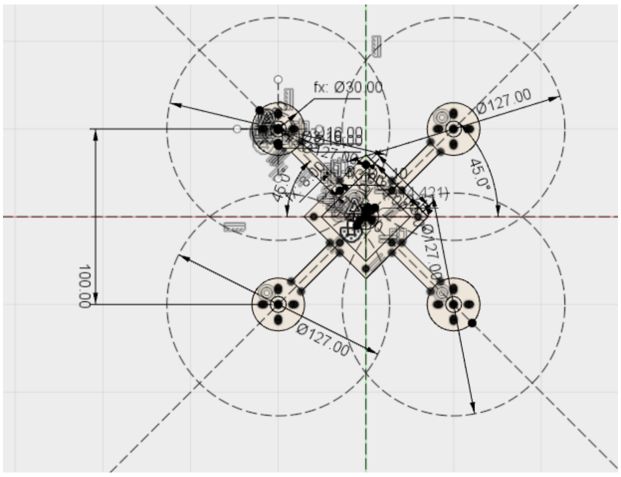

Figure 8: Final airframe design

Figure 9: Final design in flight

Total mass (without cargo) — 456 g

One can see that the value of mass is about the one obtained from theoretical investigation.

The charge in the accumulator after the 1 minute of flight was about a half of total accumulator capacity (650 mAh). This enables to estimate the current and power from accumulator, mean current is about 39 A (about 10 A per 1 motor), mean power is about 624 Watt.

5 CONCLUSION

1. Analytical investigations for the problem considered were conducted and it was found that the rational total mass is one-two masses of cargo; multi-blade propellers must be used, the optimal number of propellers is one and quadcopter gives the maximal dimension of 20% higher than for one propeller design.

2. A set of designs was made to check the analytical results and find good technical solutions. For the task investigated and components available the main solutions are: the optimal place of the cargo is above the copter; the maximal dimension of $2D$ can be made by placing the propellers in two parallel planes.

3. Copter mass for the final design coincides with the estimated one of theoretical investigation.

REFERENCES

[1] R. Deters and M. Selig. Static testing of micro propellers. In *26th AIAA Applied Aerodynamics Conference*, 2008.