Investigation on Natural Frequency and Fuselage Effect for Small UAVs Lateral Motion

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ABSTRACT

An accurate mathematical model is necessary for controlling an aircraft. Although the geometrical scale of Unmanned Aerial Vehicles (UAVs) is very small compared to the large aircrafts, they are usually designed by means of the procedure intended for large ones, and stability calculations similarly follow the same formulas. This fact can severely affect the basic assumptions of the formulas and hence it may not be suitable for UAVs. This research validates the dutch roll natural frequency of lateral motion calculated by comparing the usual methods of estimation for the manned aircraft found in references of Roskam and Ostoslavsky, and the numerical Vortex Lattice Method (VLM) program XFLR5 with experimental values of real flight. Also a study is carried out to examine the effect of fuselage on the dutch roll natural frequency to examine the possibility of neglecting it through the calculations. It is found that approximate methods for Roskam procedure is in accordance with the exact solution, and the same for Ostoslavsky. Estimation methods of Roskam (exact), Ostoslavsky and XFLR5 give good results in agreement with the experiment, while the approximate methods of Roskam underestimate the frequency. The contribution of the regular fuselage is found to be very small and it can safely be neglected.

NOMENCLATURE

ω_n	dutch roll mode natural frequency
θ	pitch angle
b	wing span
C_L	airplane lift coefficient in steady state condition
g	gravitational acceleration
I_{xx}	moment of inertia around fuselage axis
I_{yy}	moment of inertia around wing axis
I_{zz}	moment of inertia around normal to fuselage axis
L_β	roll angular acceleration per unit sideslip angle
L_p	roll angular acceleration per unit roll rate
L_r	roll angular acceleration per unit yaw rate
m	aircraft mass
N_β	yaw angular acceleration per unit sideslip angle
N_p	yaw angular acceleration per unit roll rate
N_r	yaw angular acceleration per unit yaw rate
N_{T_β}	yaw angular acceleration per unit sideslip angle (due to thrust)
S	wing reference area
U_1	cruise velocity
Y_β	lateral angular acceleration per unit sideslip angle
Y_p	lateral angular acceleration per unit roll rate
Y_r	lateral angular acceleration per unit yaw rate

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1 INTRODUCTION

In order to design a control system for an aircraft, the main step is to make a mathematical model of flight mechanics for the aircraft. The controller's accuracy depends on the accuracy of the mathematical model with respect to the physical model. Even a simple PID control could be used to make the required response if the mathematical model was accurate enough.

According to flight mechanics, if any disturbance influences the aircraft (as gust wind or control surface deflection) the stable aircraft tries to damp this disturbance and return to its initial state. This behaviour is very important for the non-manoeuvrable aircraft and the knowledge about the stability is strongly required for the aircraft design and autopilot design.

For the conventional airplane shapes it is possible to separate

the disturbed motion on the longitudinal and lateral ones.
Lateral motion can be modelled as the fourth-order equation that describes three modes: spiral, roll and dutch roll. The first two modes are first-order (corresponding to the exponential decrease or increase) and the dutch roll is second-order (corresponding to the decreasing or increasing oscillations or exponential increasing/decreasing).
In case of damped vibrations, there are two definitions for the frequency: the damped frequency and the undamped/natural frequency. In this paper, the natural frequencies are considered.
Since the UAVs (including mini and micro) are usually designed according to the procedure used for the large aircrafts, the stability calculations also commonly follow the same formulas derived for the large aircrafts, but the geometrical scale of UAVs is much smaller. So, the forces acting on the UAV change their order of power nonlinearly and some assumptions for the manned aircrafts may be not valid in this case, and new assumptions can be introduced. One of the main questions is how the formulas for these frequencies change with the scale and Reynolds number.
First attempts of these investigations were conducted previously in [1]. Among all conclusions, the main finding was the possibility of separating the equations of UAVs disturbed motion into the longitudinal and lateral motions. In [2] a test case was studied for an UAV of mass of 150 gram, wing span of 85 cm. Based on the procedure of [3] it was found that the natural frequency of the short mode of longitudinal direction is big enough compared to the long mode. Longitudinal flight modes were investigated in detail in [4]. It is found that the natural frequency of the long mode can be predicted accurately by the exact methods used in [5] and [3] but the short mode was not captured due to the high damping ratio and the testing conditions. A method was recommended to overcome the high damping ratio in [6] by shifting the center of gravity (CG). Now the dutch roll oscillation of the lateral motion is being investigated in detail.
The goal of this investigation is to understand how accurate the calculations based on the "traditional" formulas with respect to the experimental values are. In this research, an investigation is carried out on the formulas and assumptions of calculating the natural frequency of the dutch roll mentioned by J. Roskam, D. Hull, and I. Ostoslavsky and compare their results with the numerical VLM calculations from XFLR5 which is mainly designed for small UAVs then these results are compared to real measurements of the natural frequencies obtained from UAV "Sonic 185" at flight.
For big aircrafts usually a mathematical model is created to simulate its response to follow predefined action in case of small air disturbance and the response is recorded in the form of flight path angles and compared to the oscillations from flight log of flight test to validate the mathematical model and in particular the frequency. Such a method is not applicable for small UAVs because the disturbances are relatively high compared to the forces applying on the UAV, that's why another method is proposed. Instead of comparing the data of flight angles obtained from the experiment and mathematical model, the flight angles are processed to obtain the main parameters of modelling (natural frequencies) and to compare them to the theoretical results. This method seems to be more appropriate for small UAVs flying in disturbed air.
Fuselage has essential role in the aircraft as it has to carry the weight and it is also affects the aerodynamic performance specially drag. Calculating the fuselage contribution analytically is not straight forward because of the shape complexity. Since drag force doesn't have big importance in calculating the natural frequencies of the aircraft, a study is conducted to quantify the effect of modelling of the fuselage to examine its importance for the scale of UAVs.

2 Investigated UAV

The UAV used in this research is "Sonic 185" of DYNAM [7]. Aircraft parameters and geometry are measured and listed in Tables 1 and 2.
Due to the absence of data about the airfoils used in the wing and empennage, another airfoil was estimated to have nearly similar profile shape from the known ones. Estimation of the airfoil is based on measurements of the thickness to chord ratio and the position of maximum camber and searching for a similar airfoil. Given that thickness to chord ratio is 10.7% at 39%, this leads to choose the airfoil E231 of Eppler series which shows good convergence as its thickness to chord ratio is 12.3% at 39.4%. For the tail unit, NACA 0006 is used.

Property	value
Mass $[kg]$	1.183
I_{xx} $[kg{\cdot}m^2]$	0.108
I_{yy} $[kg{\cdot}m^2]$	0.065
I_{zz} $[kg{\cdot}m^2]$	0.122
Cruise velocity $[m/s]$	8
Aspect ratio	10.295
Span $[m]$	1.85
Wing area $[m^2]$	0.33
Center of mass from leading edge of root section $[m]$	0.07

Table 1: Aircraft parameters.

3 Analytical Approach

Roskam and Ostoslavsky considered this task by two different procedures using dimensional and nondimensional stability parameters as shown below.

3.1 Roskam Procedure

Roskam's procedure [5] starts from estimating the aerodynamic coefficients (see Table 3), calculating the forces acting on the aircraft, then calculates the main characteristic

Property	Wing	Horizontal Tail	Vertical Tail
Aspect ratio	10.295	4.92	2.03
Root chord $[m]$	0.205	0.125	0.2
Tip chord $[m]$	0.06	0.02	0.115
Mean chord $[m]$	0.189	0.1	0.16
Span $[m]$	1.85	0.48	0.16
Area $[m^2]$	0.33	0.046	0.03
Sweep angle from leading edge $[degree]$	6.71	18.17	23.25

Table 2: ”Sonic 185” geometry.

equation. Based on the big amount of experimental data, the coefficients are estimated taking into account many details which may increase the accumulative errors during calculations. The characteristic equation of the lateral motion [5]

Derivative	Sideslip Angle (β)	Roll Rate (p)	Yaw Rate (r)
Side Force Coeff. (C_y)	-0.17	0	0.14
Roll Moment Coeff. (C_l)	-0.113	-0.873	0.204
Yaw Moment Coeff. (C_n)	0.064	-0.127	0.124

Table 3: Aerodynamic lateral derivatives based on Roskam method.

is:

$$Ax^4 + Bx^3 + Cx^2 + Dx + E = 0 \tag{1}$$

where

$$A = U_1(1 - \bar{A}\bar{B}) \tag{2}$$

$$B = -Y_\beta(1 - \bar{A}\bar{B}) - U_1(L_p + N_r + \bar{A}N_p + \bar{B}L_r) \tag{3}$$

$$\begin{aligned} C = {} & U_1(L_pN_r - L_rN_p) + Y_\beta(N_r + L_p + \bar{A}N_p \\ & + \bar{B}L_r) - Y_p(L_\beta + N_\beta\bar{A} + N_{T\beta}\bar{A}) + U_1(L_\beta\bar{B} \\ & + N_\beta + N_{T\beta}) - Y_r(L_\beta\bar{B} + N_\beta + N_{T\beta}) \end{aligned} \tag{4}$$

$$\begin{aligned} D = {} & -Y_\beta(L_pN_r - L_rN_p) + Y_p(L_\beta N_r - N_\beta L_r \\ & - N_{T\beta}L_r) - g\cos\theta_1(L_\beta + N_\beta\bar{A} + N_{T\beta}\bar{A}) \\ & + U_1(L_\beta N_p - N_\beta L_p - N_{T\beta}L_p) - Y_r(L_\beta N_p \\ & - N_\beta L_p - N_{T\beta}L_p) \end{aligned} \tag{5}$$

$$E = g\cos\theta_1(L_\beta N_r - N_\beta L_r - N_{T\beta}L_r) \tag{6}$$

where

$$\bar{A} = I_{xz}/I_{xx} \tag{7}$$

$$\bar{B} = I_{xz}/I_{zz} \tag{8}$$

After analysis, the results showed that natural frequency (ω_n) for dutch roll is 0.59 Hz. To simplify the decomposition of modes Roskam made an approximate solution for obtaining the natural frequency directly instead of solving the main fourth order equation by linking the forces directly to the natural frequency as follows:

$$\omega_{n1} = \sqrt{N_\beta + \frac{Y_\beta N_r - N_\beta Y_r}{U_1}} \tag{9}$$

Under the assumption that $(Y_\beta N_r - N_\beta Y_r)/U_1$ is significantly less than N_β, the former equation will be:

$$\omega_{n2} = \sqrt{N_\beta} \tag{10}$$

The natural frequency in these cases are ω_{n1}=0.5 Hz and ω_{n2}=0.49 Hz respectively.

3.2 *Ostoslavsky Procedure*

Ostoslavsky [3] has derived the characteristic equation by a different method. Instead of calculating the forces, the non-dimensional aerodynamic coefficients are used directly then the coefficients of the lateral characteristic equation are obtained. Using simple geometrical parameters, the aerodynamic coefficients can be estimated simply. Results are listed in Table 4.

Derivative	Sideslip Angle (β)	Roll Rate (p)	Yaw Rate (r)
Side Force Coeff. (C_y)	-1.494	0	0
Roll Moment Coeff. (C_l)	-0.011	-1.038	-0.089
Yaw Moment Coeff. (C_n)	0.036	0.073	0.032

Table 4: Aerodynamic lateral derivatives based on Ostoslavsky method.

Ostoslavsky introduced the characteristic equation of the system as fourth order function in its eigen values as:

$$F = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0 \tag{11}$$

where the coefficients of these equations under the steady state condition are:

$$a_1 = -(0.5c_{L\beta} + \bar{c_{lp}} + \bar{c_{nr}}) \tag{12}$$

$$\begin{aligned} a_2 = {} & c_{L\beta}/2(\bar{c_{lp}} + \bar{c_{nr}}) + (\bar{c_{lp}}\bar{c_{nr}} \\ & - \bar{c_{lr}}\bar{c_{np}}) - \mu(\bar{c_{n\beta}} + \alpha\bar{c_{l\beta}}) \end{aligned} \tag{13}$$

$$a_3 = -\mu(\bar{c_{l\beta}}\bar{c_{np}} - \bar{c_{n\beta}}\bar{c_{lp}}) \tag{14}$$

$$\begin{aligned} a_4 = {} & \mu c_L(\bar{c_{l\beta}}\bar{c_{nr}} - \bar{c_{n\beta}}\bar{c_{lr}} + \tan\theta_0) \\ & (\bar{c_{l\beta}}\bar{c_{np}} - \bar{c_{n\beta}}\bar{c_{lp}})/2 \end{aligned} \tag{15}$$

where

$$\overline{c_{lp,lr,l\beta}} = \frac{c_{lp,lr,l\beta} m b^2}{4 I_{xx}} \quad (16)$$

$$\overline{c_{np,nr,n\beta}} = \frac{c_{np,nr,n\beta} m b^2}{4 I_{yy}} \quad (17)$$

$$\mu = \frac{2m}{\rho S c} \quad (18)$$

For simplification of the analysis, the following approximate equation can be used:

$$\omega_{n1} = -\frac{a_3}{\bar{c_{lp}} 2\pi\tau} \quad (19)$$

where

$$\tau = \frac{2m}{\rho S U_1} \quad (20)$$

The results of the exact method show highly-damped situation while the approximate method showed that ω_{n1}=0.556 Hz.

4 Numerical Approach

In numerical methods as Vortex Lattice Method (VLM) – as used in XFLR5 – the aircraft is divided into small panels. For each panel a combination of source, sink, and vortex is added in one quarter of the panel, and a control point is added after three quarters of the panel to achieve the no-penetration condition [8]. By solving *N* equations obtained from the *N* panels, the vortex strength is determined for each panel then the normal and tangential forces acting on the aircraft are obtained then converting them into non-dimensional coefficients. The next step is to import these values – which depend on the angle of attack and velocity – into the state space matrix and obtain the eigen values of the matrix which are a combination of natural frequency and damping ratio and they can be separated easily. XFLR5 is used because it is open source and used widely for UAV design process and also has the ability to obtain the natural frequencies and damping ratios directly.

The UAV is plotted using the measurements from Tables 1 and 2 as illustrated in Figure 1. It is noted that fuselage in this case has is not modelled.

Calculation done showed that ω_n=0.57 Hz.

5 Experimental Approach

The experiment was conducted for the steady cruise flight with some excitation for the dutch roll mode. To prevent additional disturbances, the plane was controlled in manual mode without autopilot stabilization. During the flight the aircraft was balanced so as it moves straight with constant altitude and constant velocity. Flight parameters were controlled form the ground station of Ardupilot by means of telemetry link. Flight data are obtained from the Inertial Measuring Unit (IMU) of the "ArduPilot Mega" autopilot which measure pitch, roll, and yaw angles of the aircraft and has sampling frequency of 3.7 Hz.

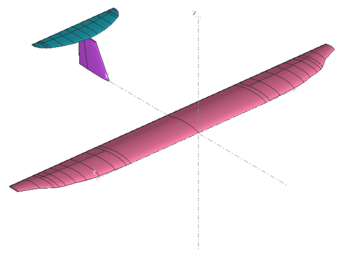

Figure 1: Sonic 185 drawing in XFLR5.

The retrieved data are processed by Fast Fourier Transform [9] once without filter and another time with filter using MATLAB. While examining the signal without filter, it is taken into account the signal first and last points have the same values to prevent aliasing.

Data are filtered by Hanning filter [10, 11] to prevent leakage in the transform [12]. Such filter is chosen for this case because:

- overcome the noise and get the mean value of the frequency,
- the exact amplitude of the frequency is not as important as the value of the frequency itself,
- the investigated signal is random and have unknown frequency,
- the vibrations are within narrow band.

For these four reasons, the most suitable filter is Hanning filter [12]. Sample of measured yaw angle is shown in Figure 2. Fast Fourier Transform is used in converting discrete time samples from time to frequency domain. After processing and filtering, the results show a freq of 0.61±0.06 Hz as shown in Figures 3–5 that show samples of the obtained results at different periods of time.

6 Fuselage Effect

Fuselage is one of the main parts of the aircraft which have direct influence on the behaviour of the aircraft towards disturbances.

Effect of the fuselage contributes in two effects: inertial forces and aerodynamic forces. Inertial forces are critical and cannot be omitted because the fuselage is essential source of weight and its inertia is essential. Aerodynamic forces applied on the fuselage has an influence on the total

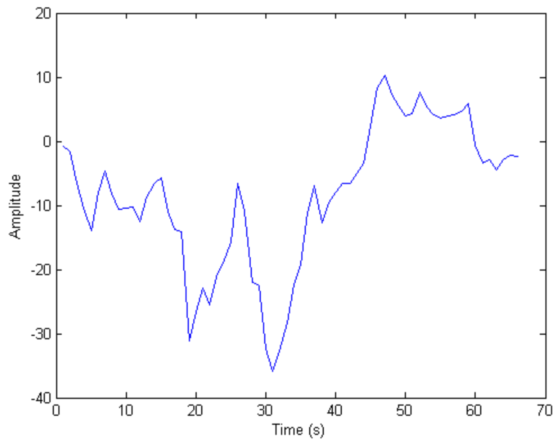

Figure 2: Sample of yaw angle recorded during flight.

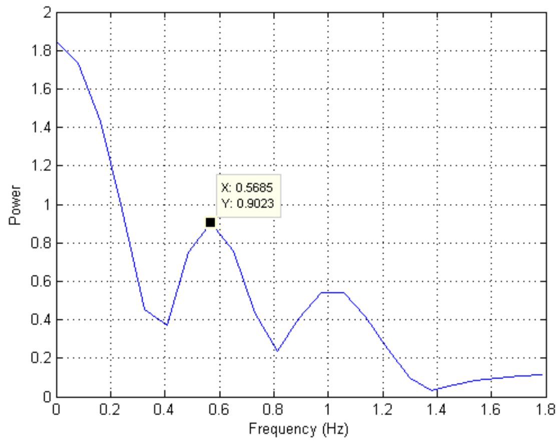

Figure 3: Sample 1 of dutch roll mode frequency.

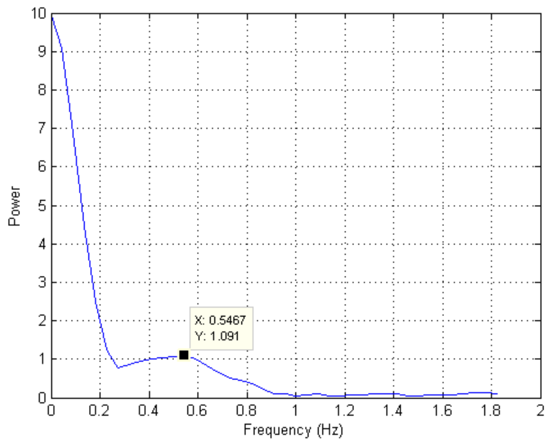

Figure 4: Sample 2 of dutch roll mode frequency.

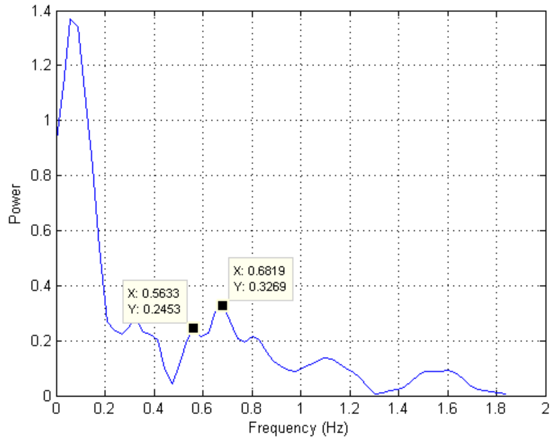

Figure 5: Sample 3 of dutch roll mode frequency.

forces and moments applied on the aircraft which determine its behaviour towards any disturbances. Calculation of aerodynamic and stability parameters for fuselage in the theoretical methods is complex and needs many information about the exact layout which is not available yet in the preliminary design phase. On the other hand, many VLM codes need much less information and an approximate layout is enough to estimate the applied forces. This conflict is the motivation to investigate the influence of the fuselage aerodynamic effect in calculation of natural frequency for the dutch roll mode. This mode is selected especially because it is quite popular that modeling of the fuselage is important is mandatory / important for lateral motion, and now this lemma is being criticized.

To investigate the fuselage aerodynamic influence on the dutch roll natural frequency, the frequency is examined by comparing two UAV cases: without and with fuselage using XFLR5. The second configuration, shown in Figure 6, is calculated in the same way explained in Section 4.

Results show that ω_{n1}= 0.582 Hz for the case with fuselage.

7 Analysis and Discussion

It is noticed that the aerodynamic coefficients estimated from the procedures of Roskam and Ostoslavsky are not matched together, for example the parameter $c_{y\beta}$ in Ostoslavsky method is ten times higher than Roskam. Though, there is good agreement in the natural frequencies. From here it is concluded that each method must use its own aerodynamic and stability coefficients.

The exact method of Ostoslavsky shows high damping ratio while the approximate solutions give results in accordance with the experimental one, which means that the assumptions used are still valid for UAVs. On the other hand, approximate

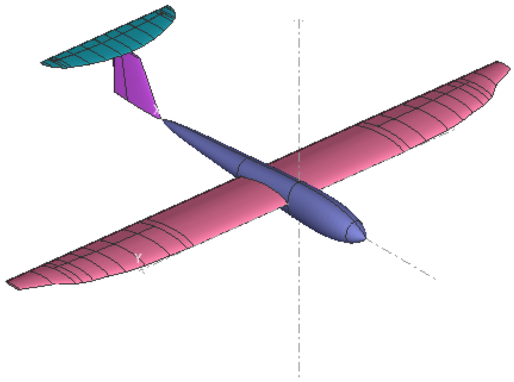

Figure 6: Sonic 185 including fuselage in XFLR5.

methods of Roskam don't achieve the required accuracy, and hence the assumptions he did are invalid for the case of small UAVs.

Calculations and experiments of this research – beside [4, 6]) – show that the frequencies for the longitudinal and lateral modes are different, so it is possible to separate the longitudinal and lateral disturbed motions for the aircrafts of the type investigated. Also the dutch roll mode can be sorted out from the whole lateral motion.

By comparing the results of the exact method of Roskam, approximate method of Ostoslavsky and XFLR with the experiment, it is notable that these methods give result in the same order of the actual one as shown in Table 5 while the approximate methods of Roskam are not accurate enough. This means that a set of methods valid for the large aircrafts can be implemented to the smaller ones (for lower Re numbers and small geometric scales).

Method	Frequency (Hz)
Roskam – exact	0.59
Roskam – approx (1)	0.50
Roskam – approx (2)	0.49
Ostoslavsky – exact	–
Ostoslavsky – approx	0.56
XFLR5 without fuselage	0.57
XFLR5 with fuselage	0.58
experiment	0.61 ± 0.06

Table 5: Results of dutch roll natural frequency using the different methods.

Formulas and experimental data show that the value of damping ratio is very close to the one corresponding to aperiodical motion. In this case to define the mode realizing (periodic or aperiodic) rather precise values of necessary parameters are required. As we can't grantee the absolute precision of parameters' values used in the calculations this can explain the fact that exact Otoslavsky formula predict overdamped (aperiodic motion) while approximate Ostoslavsky formula gives periodic damped motion.

Considering the fuselage effect on the dutch roll frequency, the difference between the cases with and without fuselage is 0.028 Hz, which is can be neglected compared to the uncertainty of the experiment (0.06 Hz). This means that absence of conventional fuselage will not affect the accuracy severely and it can be neglected while studying the natural frequency of the lateral mode.

8 Conclusion

This research validates the dutch roll natural frequency of lateral motion calculated by the usual methods of estimation for the manned aircraft found in the references of Roskam and Ostoslavsky, and the numerical VLM program XFLR5 with experimental values of real flight. It is found that exact and approximate methods of Ostoslavsky, exact method of Roskam, and XFLR5 estimate the frequency within range of the experimental results while the approximate methods of Roskam underestimates the frequency and hence the assumptions used are not valid in case of small UAVs. Results from analytical methods are valid only for the aerodynamic coefficients defined in the same procedure.

The methods for the large aircraft dynamics of the disturbed motion can be implemented to the smaller aircrafts and lower Re numbers while the assumptions must considered carefully taking into consideration that some stability coefficients cannot be neglected as in the case of large aircraft.

The role of fuselage in natural frequency estimation is examined and it is found that absence of conventional fuselage will not affect the accuracy severely and it can be neglected.

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