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Relative Localization for Quadcopters using Ultra-wideband Sensors

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ABSTRACT

In this paper a relative localization (RL) system for quadcopters based on ultra-wideband (UWB) ranging measurements is proposed. To achieve the relative localization, UWB modules are installed on the quadcopters to actively measure distances and exchange data package to a hovering quadcopter which is equipped with an identical UWB module. Since instantaneous distance-only measurements cannot provide enough information for relative position estimate, a combination of nonlinear and linear trajectory is ultilized to fulfill the RL in this paper. To reduce the estimate error, the heading in the linear path phase is designed and chosen online. This intial relative position estimate produced by the algorithm will be further fed to the flight control loop to aid the navigation of the quadcopters. Flight tests have been conducted to validate the performance of UWB based relative localization algorithm.

1 INTRODUCTION

Micro unmanned aerial vehicles (UAVs) are promising to play more and more important roles in both civilian and military activities. Currently, the navigation of UAVs is critically dependent on the localization service provided by the Global Positioning System (GPS), which suffers from the multipath effect [1] and blockage of line-of-sight, and fails to work in an indoor, forest or urban environment [2].

In certain environments such as forest and multi-functional office, even though UAV can execute self-localization and navigation through LiDAR sensor, camera, etc. However, for a long-time flight, the percent of position drift from simultaneous localization and mapping (SLAM) system can degrade significantly so that it can no longer support autonomous flight. Theoretically, when UAVs are close to each other, the localization solution for UAVs can be improved by using relative position estimates. Hence, a relative localization system can be useful to aid the navigation for UAVs in a blind environment.

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The study on the cooperative localization can be dated back to as early as the 90s of last century, when in [3] a repeated move-and-stop actions were employed with static robots serving as landmarks to localize the moving robots. A huge body of works has flourished after 2000, and can be roughly classified into two categories [4, 5] as based on whether fusion or optimization method is used.

The majority of cooperative localization literature is dedicated to fusion method, either by particle filter or by extended Kalman Filter. By assuming that the location of each robot is independent from the others', [6] applied the method of non-parametric belief propagation to marginalize the joint distribution as the estimate of each agent. By carefully examining the timing of marginalization in a Markov network, [7] was able to account for the cross correlation with a delayed estimate for each robot in the team. Later it was extended in [8] in a more convenient information filter form, with a central processing center dealing with the relative measurements. The fusion by covariance intersection method is considered in [9] and [4]. Based on discrete-time range measurements from an agent to a source, [10] addresses the position estimation problem of a drifting source relative to an agent in three-dimension by designing a Kalman filter with globally exponentially stable error.

The other works are based on the optimization. Assuming that in each time step the motions and observations are independent among different robots, [11] obtained the estimate by solving a MLE problem. For the distributed implementation, it suggested an optimum seeking method by fixing the neighbor's position, and then exchanging the suboptimal estimate for another round of optimization. [12] considered a MAP formulation, and achieved the cooperative localization by distributed conjugate gradient method. When the measurements are relative positions, Jacobian algorithm was used to solve the distributed localization in [13]. Moving horizon estimation (MHE) and convex optimization were applied to perform multirobot localization with constraints and unknown initial poses in [5].

Compared with the global and relative position information, range measurement is relatively easy to obtain among multi-UAVs. In this case, recently many researchers focus on range based relative localization [14, 10]. Typically the distance from a moving UAV to its neighbors can be estimated from RF signals. Among different types of RF ranging techniques, UWB ranging technology is robust to multipath and non-line-of-sight (NLOS) effects, and can achieve a centimeter level ranging error. The ultra wide bandwidth enables

UWB modules to avoid the interface with other types of RF signals such as WiFi and remote control signals in the flight environment. Besides, the ultra-short duration pulses can be exploited to simultaneously obtain precise range and data information, which provides extra information from neighbors for relative localization.

In this paper, we establish a relative localization system for quadcopters based on UWB range measurements. To achieve the relative localization, an UWB module is installed on the quadcopter to actively send ranging requests and data package to neighboring UAVs. From a practical and computation point of view, only a part of distance and neighbor's height measurements go through calibration and relative localization algorithm. Flight tests with three UAVs have been conducted to evaluate our developed relative localization algorithms.

In the following section, we will formulate the relative localization (RL) problem. In developing a solution to the problem, we begin to examine how to estimate the relative position with smaller errors in a dynamic agent group and propose an UWB based relative localization algorithm (see section 2). After that, the demonstration of the proposed relative localization for 3 UAVs in a woods near environment is presented in section 3. We conclude our work in section 4.

2 UWB BASED RELATIVE LOCALIZATION

2.1 *Problem Formulation*

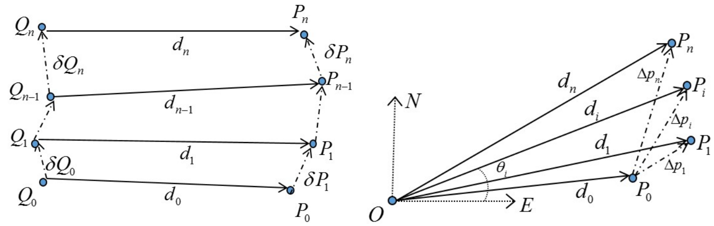
For each UAV in the team, relative localization aims to determine the position of the neighboring UAV relative to itself. If the bearing and distance of the neighbor can be obtained, then the relative localization is readily achieved. However, the relative bearing is quite difficult to achieve when the neighbor is located outside the operational range of camera. On the other hand, the distance measurement can still be stably achieved with the help of UWB. In this case, a possible method is to combine the distance measurements and the motion displacements for estimating the relative location. As shown in the Figure 1(a), for the two UAVs P and Q, relative localization aims to estimate the vector $\overrightarrow{P_iQ_i}$ from the displacements $\overrightarrow{\delta P_i}$ and $\overrightarrow{\delta Q_i}$, as well as the distances d_i, $i = 1, \ldots, n$. In this paper, we shall focus on the case when one of the UAVs is static, say Q in Figure 1(b). Specifically, we shall first discuss the proposed localization method under the assumption that the displacement $\overrightarrow{\delta P_i}$ can be accurately measured, and then revise the corresponding method to include the error in $\overrightarrow{\delta P_i}$.

2.2 *Direct Estimation by Trilateration*

2.2.1 Algorithm

Denote $\overrightarrow{QP_0} = (x, y)$ and $\overrightarrow{P_0P_i} = (\Delta x_i, \Delta y_i)$, then we have

$$d_i = \sqrt{(x + \Delta x_i)^2 + (y + \Delta y_i)^2}, i = 0, 1, \ldots, n. \quad (1)$$



(a) RL of moving UAVs (b) RL from static UAV

Figure 1: Relative localization for two UAVs

In case of ranging error of d_i's, we can estimate (x, y) by solving the nonlinear least square problem

$$\begin{aligned} &\min \sum_{i=0}^{n} (d_i - \sqrt{(x + \Delta x_i)^2 + (y + \Delta y_i)^2}) \\ &= \min \sum_{i=0}^{n} (d_i - f_i(x, y)). \end{aligned} \quad (2)$$

The solution to the above problem is usually achieved by the Gauss-Newton method, which is based on the linear approximation of f_i around the current estimate $(\hat{x}, \hat{y})$ as

$$\begin{aligned} f_i(x, y) &\approx f_i(\hat{x}, \hat{y}) \\ &+ \frac{1}{f_i(\hat{x}, \hat{y})} (\hat{x} + \Delta x_i, \hat{y} + \Delta y_i) \cdot (x - \hat{x}, y - \hat{y}). \end{aligned} \quad (3)$$

Replacing the approximation (3) in (2), problem (2) can be transformed into a linear least square problem whose normal equation is given by

$$\begin{pmatrix} d_0 - \hat{f}_0 \\ d_1 - \hat{f}_1 \\ \vdots \\ d_n - \hat{f}_n \end{pmatrix} = \begin{pmatrix} \hat{x}/\hat{f}_0 & \hat{y}/\hat{f}_0 \\ \hat{x}_1/\hat{f}_1 & \hat{y}_1/\hat{f}_1 \\ \vdots & \vdots \\ \hat{x}_n/\hat{f}_n & \hat{y}_n/\hat{f}_n \end{pmatrix} \begin{pmatrix} x - \hat{x} \\ y - \hat{y} \end{pmatrix}, \quad (4)$$

or $\Delta d = J \Delta p$ where $\hat{x}_i = \hat{x} + \Delta x_i$ and $\hat{y}_i = \hat{y} + \Delta y_i$. Now we have

$$\Delta p = \begin{pmatrix} x - \hat{x} \\ y - \hat{y} \end{pmatrix} = (J'J)^{-1} J' \Delta d \quad (5)$$

The newly obtained (x, y) can be treated as the new estimate to update J and Δd for a new normal equation. The iteration will stop until $|x - \hat{x}| < 0.01$ and $|y - \hat{y}| < 0.01$ and the problem (2) is solved.

However, we still need an initial guess to solve (2), which comes from the solution of a related least square problem dealing with the squared distances of (1). Actually, squaring both sides of (1) and subtracting the first equation $d_0^2 =$

x^2+y^2 from the other n equations, we get the following linear system

$$2\begin{bmatrix}\Delta x_1 & \Delta y_1\\ \Delta x_2 & \Delta y_2\\ \vdots & \vdots\\ \Delta x_n & \Delta y_n\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}d_1^2-d_0^2-(\Delta x_1^2+\Delta y_1^2)\\ d_2^2-d_0^2-(\Delta x_2^2+\Delta y_2^2)\\ \vdots\\ d_n^2-d_0^2-(\Delta x_n^2+\Delta y_n^2)\end{bmatrix} \quad (6)$$

or $2A\begin{bmatrix}x\\ y\end{bmatrix}=D$. Now an initial guess of $\overrightarrow{QP_0}$ can be given by

$$\begin{bmatrix}x\\ y\end{bmatrix}=\frac{1}{2}(A'A)^{-1}A'D. \quad (7)$$

Remark 2.1. *Although the equation* (6) *is able to give an exact solution when there is no ranging error, the introduction of squared distances would increase the error of calculation and only achieves a rough estimate. In fact, if the range measurement* d *is a Gaussian random variable with distribution* $N(\bar{d},\sigma^2)$*, then the variance of* d^2 *is given by* $4\bar{d}^2\sigma^2+2\sigma^4$ *dependent on* $\bar{d}^2$*. On the other hand,* (4) *employs the original measurements as the input and a rough initial guess is able to guarantee the convergent solution in a few iterations. Hence we employ the rough estimate from* (6) *as the initial guess of* (2) *and obtain the estimate from* (4)*.*

2.2.2 Error Analysis

From (5) the error covariance of the solution of (2) can be approximated by $E\left[\Delta p'\Delta p\right]=\sigma^2(J'J)^{-1}$, where $E[\cdot]$ denotes the expectation of $\cdot$, if the ranging error is assumed to be independent from each other and has a variation of σ^2. Noticing that each row of J is a normalized vector, we can rewrite J as

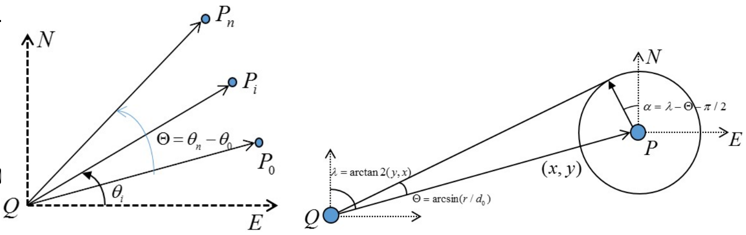
$$J=\begin{bmatrix}\cos\theta_0 & \sin\theta_0\\ \cos\theta_1 & \sin\theta_1\\ \vdots & \vdots\\ \cos\theta_n & \sin\theta_n\end{bmatrix}=\begin{bmatrix}c & s\end{bmatrix} \quad (8)$$

and obtain $(J'J)^{-1}$ as

$$(J'J)^{-1}=\frac{1}{\det(J'J)}\begin{bmatrix}s's & -c's\\ -c's & c'c\end{bmatrix}.$$

Now the mean squared error is given by the trace of $\sigma^2(J'J)^{-1}$ as

$$MSE=\frac{\sigma^2(n+1)}{\det(J'J)} \quad (9)$$



(a) Geometry of J (b) Heading to maximize Θ

Figure 2: Heading diagram

where $n+1$ is the sample size. In the meanwhile, direct computation shows that

$$\begin{aligned}\det(J'J)&=\sum_{i\neq j}\cos^2\theta_i\sin^2\theta_j-\sum_{i\neq j}\cos\theta_i\sin\theta_i\cos\theta_j\sin\theta_j\\ &=\sum_{i\neq j}(\cos^2\theta_i\sin^2\theta_j-\cos\theta_i\sin\theta_i\cos\theta_j\sin\theta_j)\\ &=\sum_{i\neq j}(\cos\theta_i\sin\theta_j-\cos\theta_j\sin\theta_i)^2\\ &=2\sum_{0\leq i<j\leq n}\sin^2(\theta_i-\theta_j),\end{aligned} \quad (10)$$

where each addend is the area spanned by the unit vectors $[\cos\theta_i\ \ \sin\theta_i]$ and $[\cos\theta_j\ \ \sin\theta_j]$. If we assume that the UAV can only obtain the accurate displacement over a short distance r and $r<<||\overrightarrow{QP_0}||$ (namely $(\Delta x_i)^2+(\Delta y_i)^2<<x^2+y^2$ for each i), then $\det(J'J)\approx 2\sum_{0\leq i<j\leq n}(\theta_i-\theta_j)^2$. Now the MSE is only dependent on the angles spanned by the corresponding unit vectors, and aligning each P_i along a straight line would not affect the localization accuracy. Therefore, we may let the UAV fly along a straight line during the relative localization. Moreover, it's reasonable to assume that each small increment of θ is equal, or $\theta_{i+1}-\theta_i=\omega$. In this case, $\det(J'J)\approx 2\sum_{0\leq i<j\leq n}(i-j)^2\omega^2=(\frac{n^4}{6}+\frac{2n^3}{3}+\frac{5n^2}{6}+\frac{n}{3})\omega^2$ and

$$MSE\approx\frac{6\sigma^2}{(n^4+4n^3+5n^2+2n)\omega^2}=\frac{6\sigma^2}{(n^2+4n+5+\frac{2}{n})\Theta^2}$$

where $\Theta=\theta_n-\theta_0$. We can see that a major reduction of the mean squared error can be from an increase of Θ. In light of this observation, the UAV is to move along the line which maximizes Θ, or the angle $\angle P_0QP_n$ as in Figure 2(a). We can see that the maximum $\Theta=\arcsin(r/d_0)$ if the UAV is to move a short distance of r for the relative localization as shown in Figure 2(b). Also, the heading in the North-East frame can be found as $\alpha=\lambda-\Theta-\frac{\pi}{2}$, where $\lambda=\arctan 2(y,x)$.

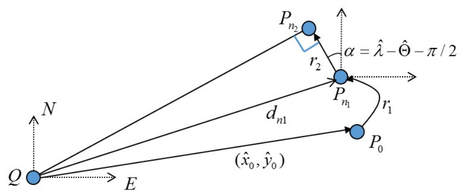


Figure 3: Two phases in the relative localization

2.2.3 Application to Relative Localization

Now we can apply the error analysis to the relative localization as follows. Actually, a linear path is not enough to uniquely determine the vector $\overrightarrow{QP_0}$ due to the ambiguity, while a nonlinear path nails it. Therefore, we propose the relative localization as a combination of a nonlinear path and a linear path. To be specific, assume that the UAV is to fulfill the relative localization in a short distance r and it first moves along some nonlinear path of fixed length r_1. After the nonlinear phase, we can actually get a rough estimate of (x_0, y_0) as $(\hat{x}_0, \hat{y}_0)$, as well as that of $\overrightarrow{QP_{n_1}}$ as $(\hat{x}_{n_1}, \hat{y}_{n_1})$. As suggested from the last subsection, we need to move along some direction by a length of $r_2 = r - r_1$ such that $\angle P_0 Q P_n$ reaches the maximum. Obviously the heading should be chosen as $\alpha = \hat{\lambda} - \Theta - \frac{\pi}{2}$, where $\hat{\lambda} = \arctan 2(\hat{y}_{n_1}, \hat{x}_{n_1})$, $\Theta = \arcsin(r_2/d_{n_1})$. These two phases are also depicted in Figure 3.

2.3 *Inclusion of Displacement Error in the Algorithm*

This section evaluates the performance of the proposed relative localization algorithms considering that both the distance measurements and the odometry estimates are subject to noise. Specifically we assume the range measurements of each UAV are subject to independent zero-mean Gaussian noise and the odometry estimates are subject to two independent sources of noise.

To apply the direct estimation method, we still need to account for the displacement error, which can be solved by the following EKF. In fact, the system model can be simply modeled as follows:

$$\begin{aligned} x(k+1) &= x(k) + \Delta x(k) + \zeta_x(k), \\ y(k+1) &= y(k) + \Delta y(k) + \zeta_y(k), \\ d(k) &= \sqrt{x^2(k) + y^2(k)} + \eta(k), \end{aligned} \tag{11}$$

where $(x(k), y(k))$ denotes the relative position of the moving UAV at the time step k, $(\Delta x(k), \Delta y(k))$ is the displacement, $d(k)$ is the UWB ranging, and $\zeta_x(k), \zeta_y(k), \eta(k)$ are the corresponding noises. With the initial estimate given by the direct trilateration, we can apply the EKF for relative localization.

With Figure 3, we summarize the relative localization of the moving UAV P from the static UAV Q as below:

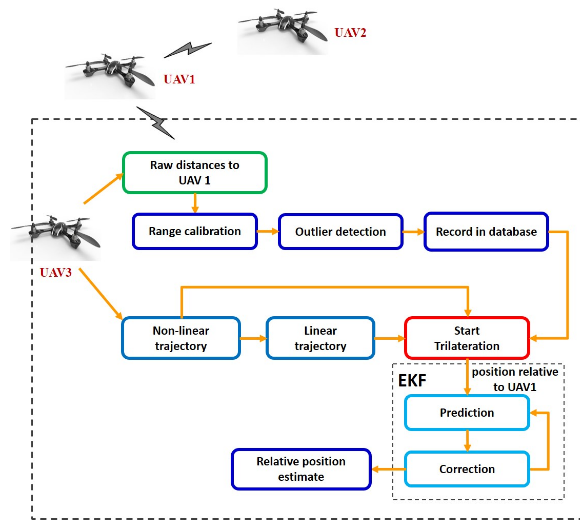


Figure 4: UWB-based relative localization workflow

I (1) P moves along a nonlinear path of fixed length r_1 to the point P_{n_1} and collect the distance measurements d_i and displacement $(\Delta x_i, \Delta y_i)$ in the NE frame, $i = 0, \ldots, n_1$.
(2) Obtain estimate of $\overrightarrow{QP_{n_1}}$ as $(\hat{x}_{n_1}, \hat{y}_{n_1})$ by trilateration algorithm described in section 2.2.1. Choose heading in the next phase by $\alpha = \hat{\lambda} - \Theta - \frac{\pi}{2}$, where $\hat{\lambda} = \arctan 2(\hat{y}_{n_1}, \hat{x}_{n_1})$, $\Theta = \arcsin(r_2/d_{n_1})$ and r_2 is the flight distance in the next phase.

II (1) With the heading of α, P moves along a linear path of fixed length r_2 to the point P_n and collect the distance measurements d_i and displacement $(\Delta x_i, \Delta y_i)$ in the NE frame, $i = 0, \ldots, n$.
(2) Obtain estimate of $\overrightarrow{QP_0}$ as $(\hat{x}_0, \hat{y}_0)$ by trilateration algorithm described in section 2.2.1, as well as the error covariance matrix $M = \sigma^2 (J'J)^{-1}$.

III The initial guess $(\hat{x}_0, \hat{y}_0)$ and M will be used to initialize the EKF based on the model (11). d_i and $(\Delta x_i, \Delta y_i)$ are used to generate the final estimate of $\overrightarrow{QP_n}$.

3 Flight Tests with Quadcopters

To validate the proposed UWB-based relative localization approach, flight tests of 3 UAVs have been conducted in like-forest environments. The detailed hardware configuration is depicted in [15]. Below, we shall discuss the experiments in details.

3.1 *UWB based Relative Localization Workflow*

Figure 4 illustrates the workflow of the UWB based relative localization system. To achieve the range-based relative localization, all of the UAVs are equipped with the same UWB module. As the two-way time of flight ranging method described in [15], the UWB module on a hovering quadcopter actively sends ranging requests to neighboring UAVs for distance measurement and communication. Once a distance is obtained by UAV2 and UAV3, it will be calibrated by linear regression where the calibration parameters are determined by a series of experiments in different environments[15]. The calibrated range then goes through the outlier detection before it is stored in the database. To reduce the computation and avoid excessive repetition of similar data, only the selected distance measurements and neighbor's information are recorded and this selection depends on the movement of each UAV. Note that trilateration is applied to trigger the algorithm and its output serves as the initial state estimate for EKF. The follow-up localization is sustained by EKF in a recursive way.

Each part of our UWB based relative localization system is elaborated in section 2.2 and 2.3. Path direction during the linear phase is analyzed in section 2.2.3

3.2 UWB based Relative Localization for 3 UAVs Flight

The location of our test site was closen at Dover in Singapore with a area of 30×40 m. To validate the performance of our proposed relative localization, the ground truth reference for calibration is necessary. Since that, a baseline, which is parallel to a ditch with 233 degrees with respect to the north, was selected as our reference. The UAVs are aimed to navigate to or hover over the relative targets we set preliminarily (yellow flags on the ground in Figure 7) and this performance will demonstrate the availability of UWB based relative localization. Some experimental details will be introduced as following.

UAV1 was set in the middle of UAV2 and UAV3 initially with 20 meters away from them as shown in Figure 5. Once UAVs took off, UAV1 hovered over the start point and UAV2 and UAV3 flied along a preset non-linear trajectory to collect distance data and generated initial relative position estimates, namely phase I. During phase II, UAV2 and UAV3 flied as a linear trajectory to improve the accuracy of the relative position estimates by using the proposed trilateration algorithm. Then in phase III, UAV2 and UAV3 flied directly to their own desired targets (the red stars in Figure 5) set in relative coordinates and the video shot during 3 UAVs flight is shown in Figure 6. Finally all of these 3 UAVs hovered over their own targets with reference to the landmarks (yellow flags on the ground in Figure 7) measured by tape measure before flight test. It can be seen from Figure 7, once accomplishing the entire phases UAV2 and UAV3 hovered over their own landmarks with the position error less than 1 meter (Figure 8).

4 CONCLUSION AND FUTURE WORK

The relative localization strategy proposed in this paper utilizes UWB radios to estimate the relative position of moving UAVs to a static one over a short distance. The error analysis of the trilateration method is employed to reduce

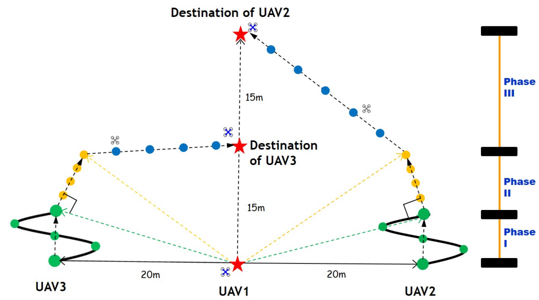


Figure 5: Demo scenario at Dover

Figure 6: Flight video shot of 3 UAVs at Dover.

Figure 7: Performance of the relative localization for 3 UAVs.

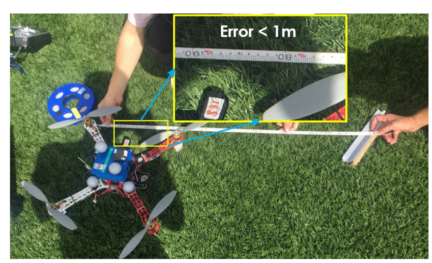


Figure 8: Accuracy of the relative localization measured after landing.

the localization error. Basically, the UAV is first to move along some nonlinear path for a rough estimate, based on which a heading will be chosen for the UAV to move along a straight line aiming to improve the localization error. The rangings and displacements over the course are first fed to the nonlinear least squares algorithm for an initial guess, which serves to initialize the EKF. The proposed trilateration algorithm and EKF cooperate to achieve highly precise relative position estimate for autonomous flight of the quadcopters. Flight tests have been conducted to validate the performance of the proposed UWB-based relative localization, and the results demonstrate its capability of providing accurate relative position for the quadcopters.

Future works will be steered to the case of relative localization when all the three UAVs are moving simultaneously. Integrated with certain self-localization algorithms, a more robust and accurate localization technique for long term flight in GPS denied environments can be investigated. Besides, UWB based UAV swarming may be explored and demonstrated in the future.

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