Numerical Investigation of a Proof-of-Concept Rotor in Martian Atmosphere

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ABSTRACT

This paper presents 2-D Blade Element Momentum Theory predictions and 3-D Reynolds-averaged Navier-Stokes calculations for a proof-of-concept rotor in Martian atmosphere. The rotor that operates on Mars will experience a unique low Reynolds number and high Mach number environment, thus it presents a new challenge. The sectional aerodynamic characteristics of Eppler 387 airfoil are obtained from a 2-D Reynolds-Averaged Navier-Stokes solver. Based on the sectional aerodynamic characteristics, the thrust coefficient and Figure of Merit of rotor are initially predicted by Blade Element Momentum Theory. Then, the rotor with a collective pitch of $10^\circ$ is investigated using a 3-D Reynolds-Averaged Navier Stokes solver. The results show a clear discrepancy between 2-D predictions and 3-D calculations due to 3-D rotation effect, such as stall modification and root vortices. In addition, the 3-D calculations are not sensitive to the usage of turbulent model or transition model when compared to 2-D predictions.

1 INTRODUCTION

During the past fifty years, planetary exploration has progressed greatly, especially on Mars. The rovers have been launched successfully and provided invaluable information of Mars [1]. Recently, rotary wing vehicle has attracted the attention of the Mars exploration community [2–5]. In contrast to rover, rotary wing vehicle has greater range, speed and field of view. Moreover, it will not be restricted by the unprepared Martian terrain. The concept and feasibility of Martian rotary wing vehicle have been demonstrated by the Army/NASA Rotorcraft Division at NASA Ames Research Center. A NASA Mars rotor prototype was initially proposed based on a coaxial configuration [6]. The study mainly described numerical methods for use in Mars rotor design.

Martian atmosphere proposes unique design problems for rotary wing vehicle, as seen in Table 1. On Mars, the gravity is 0.4 times that of gravity on Earth while the pressure is 0.008 times that of the pressure on Earth. According to Martian density and viscosity, the Reynolds numbers on Mars is 0.02 times what is on Earth. Besides, the speed of sound on Mars is 0.7 times that of the speed of sound on Earth. Therefore, producing the required lift will generate a very high tip Mach number in Martian atmosphere. Finally, the rotor that operates on Mars will suffer a unique low Reynolds number and high Mach number environment, which proposes unconventional design requirement.

Table 1: Comparison of Mean Atmospheric Properties.

<table>
<thead>
<tr>
<th></th>
<th>Earth</th>
<th>Mars</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gravity, m/s$^2$</td>
<td>9.81</td>
<td>3.66</td>
</tr>
<tr>
<td>Pressure, Pa</td>
<td>1.0135e5</td>
<td>790.53</td>
</tr>
<tr>
<td>Temperature, K</td>
<td>288.16</td>
<td>210.56</td>
</tr>
<tr>
<td>Density, kg/m$^3$</td>
<td>1.225</td>
<td>0.0167</td>
</tr>
<tr>
<td>Viscosity, kg/(ms)</td>
<td>1.789e-5</td>
<td>1.289e-5</td>
</tr>
<tr>
<td>Gas constant, J/kg·K</td>
<td>287</td>
<td>192</td>
</tr>
<tr>
<td>Adiabatic coefficient</td>
<td>1.4</td>
<td>1.29</td>
</tr>
<tr>
<td>Speed of sound, m/s</td>
<td>320</td>
<td>230</td>
</tr>
<tr>
<td>Atmospheric Gases</td>
<td>78%N$_2$</td>
<td>95%CO$_2$</td>
</tr>
<tr>
<td></td>
<td>21%O$_2$</td>
<td>2.7%N$_2$</td>
</tr>
</tbody>
</table>

The first Chinese Mars obiter is Yinghuo 1, as shown in Figure 1. The scientific objectives of Yinghuo 1 were:

Figure 1: Yinghuo 1 Mars obiter.

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• To conduct investigation of plasma environment and magnetic field;
• To study ion escape processes and mechanisms;
• To perform ionosphere occultation measurement;
• To observe sandstorms on the Martian surface.

In 2011, Yinghuo 1 obiter and Fobos-Grunt spacecraft were launched from Baikonur Cosmodrome, Russia. However, the subsequent rocket burns failed and Yinghuo 1 could not enter Mars orbit. In 2016, the head of China National Space Administration (CNSA) announced that China will launch a new Mars probe in 2020 and start Mars exploration. Under this circumstance, Martian rotary wing vehicle is proposed for the potential exploration mission. Taking NASA rotor prototype as a reference, a small-scale proof-of-concept rotor is proposed based on the stowed dimension of Yinghuo 1 (length 0.75m, width 0.75, and height 0.6m). Table 2 lists the specifications of the proof-of-concept rotor. A conventional low Reynolds number airfoil Eppler 387 is selected for the proof of concept rotor. Notice that 40% root cut-out is to simulate blade telescoping required for transport and storage. The typical helicopters operating on Earth usually have tip Mach number of 0.8, but parts of the flow can accelerate to Mach 1. In Martian atmosphere, tip Mach number 0.65 is chosen to keep the blade far from sonic conditions.

Table 2: Proof-of-concept rotor description.

<table>
<thead>
<tr>
<th>Number of blades</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor radius</td>
<td>0.25m</td>
</tr>
<tr>
<td>Blade root cut-out</td>
<td>40%</td>
</tr>
<tr>
<td>Airfoil</td>
<td>Eppler 387</td>
</tr>
<tr>
<td>Mach number at R</td>
<td>0.65</td>
</tr>
<tr>
<td>Reynolds number at R</td>
<td>5,900</td>
</tr>
<tr>
<td>Mach number at 0.75R</td>
<td>0.48</td>
</tr>
<tr>
<td>Reynolds number at 0.75R</td>
<td>4,400</td>
</tr>
<tr>
<td>Blade chord</td>
<td>0.03</td>
</tr>
<tr>
<td>Linear twist rate</td>
<td>+2.4° to −2.4°</td>
</tr>
</tbody>
</table>

In present study, a Blade Element Momentum Theory (BEMT) model is employed to analyze the proof-of-concept rotor. The sectional aerodynamic characteristics used for BEMT is calculated from a 2-D Reynolds-Averaged Navier-Stokes (RANS) solver. Then, a 3-D Reynolds-Averaged Navier Stokes solver is used to investigate the rotor with a collective pitch of 10°. This paper aims at exploring the numerical methods for future Mars rotor design and assessment.

2 2-D BEMT Predictions

The BEMT is a method that combines the blade element theory and one dimensional momentum theory. BEMT allows the inflow distribution along the blade to be estimated with the assumption that the elements in spanwise direction have no mutual effects on each other. Twist distribution, chord distribution, airfoil characteristics and operation condition are needed for a BEMT analysis. The rotor disk area can be discretized into successive annuli with an area

\[ dA = 2\pi y dy \]  

The axial climb velocity \( V_c \) can be generalized by free-stream velocity \( V_\infty \), say \( V_c = V_\infty \). Based on one dimensional momentum theory, the incremental thrust on the rotor annulus can be calculated as the product of the mass flow rate through the annulus and twice induced velocity at the section. The incremental thrust on the annulus is:

\[
dT = 2\rho(V_\infty + v_i)v_i dA = 4\rho \sigma_0 (V_\infty + v_i) v_i y dy, \tag{1}
\]

where \( \rho \) is the density of the fluid and \( v_i \) is the induced velocity. Using non-dimensional quantities, Equation 1 becomes:

\[
dC_T = \frac{2\rho(V_\infty + v_i)v_i dA}{\rho(\pi R^2)(\Omega R)^2} = 4(V_\infty + v_i) \sigma_0 \left( \frac{v_i}{\Omega R} \right) \left( \frac{y}{R} \right) d\left( \frac{y}{R} \right), \tag{2}
\]

where \( R \) is the blade radius and \( \Omega \) is the rotational speed. The incremental thrust coefficient can be simply written as:

\[
dC_T = 4\lambda \lambda_\infty r dr = 4\lambda (\lambda - \lambda_\infty) r dr, \tag{3}
\]

where \( \lambda \) is the total inflow ratio, \( \lambda_\infty \) is the induced inflow ratio, \( \lambda_\infty \) is the tip speed ratio, \( r \) is the non-dimensional radial location and \( dr \) is the non-dimensional length of each element.

Tip vortex results in an increased inflow at blade tip, reducing the effective angles of attack there. Therefore, the lift produced at the outer sections of each blade gradually decreases with the radius increasing. Blade tip effect can be considered in BEMT model using the Prandtl tip loss function. The correction factor \( F \) is expressed by:

\[
F = \frac{2}{\pi} \cos(e^{-f}), \tag{4}
\]

and \( f \) is defined by:

\[
f = \frac{N_b}{2} \left( \frac{1 - r}{r\phi} \right), \tag{5}
\]

\[
\phi = \frac{\lambda(r)}{r}, \tag{6}
\]

where \( N_b \) is the number of blades and \( \phi \) is the inflow angle. The correction factor \( F \) can also be included in the one dimensional momentum model to change the flow velocity through the annulus. Hence, Equation 3 becomes:

\[
dC_T = 4\lambda \lambda_\infty r dr. \tag{7}
\]

Using blade element theory, the incremental thrust coefficient on the annulus is:

\[
dC_T = \frac{1}{2} \sigma C_l r^2 dr = \frac{\sigma C_l r^2 (\theta r^2 - \lambda r)}{2} dr, \tag{8}
\]

where \( \sigma \) is the local solidity (\( \sigma = \frac{N_b c}{\pi R} \), \( c \) is the local chord of blade), \( C_l \) is the lift coefficient, \( C_{l_{\alpha}} \) is the 2-D lift-curve-slope of airfoil and \( \theta \) is the local pitch angle. The local pitch angle \( \theta \) is defined by:

\[
\theta = \theta_0 + \theta_{tw}, \tag{9}
\]
where $\theta_0$ is the collective pitch angle and $\theta_{tw}$ is the twist angle. Equating the incremental thrust coefficients from momentum theory (Equation 7) and blade element theory (Equation 8) gives:

$$4F \lambda (\lambda - \lambda_\infty) r dr = \frac{\sigma C_{l_\alpha}}{2} (\theta r^2 - \lambda r) dr.$$  \hspace{1cm} (10)

Rearranging Equation 10 in terms of $\lambda$, the canonic form is obtained:

$$\lambda^2 + \left( \frac{\sigma C_{l_\alpha}}{8F} - \lambda_\infty \right) \lambda - \frac{\sigma C_{l_\alpha}}{8F} \theta r = 0.$$  \hspace{1cm} (11)

This quadratic equation in $\lambda$ has the positive solution:

$$\lambda(r, \lambda_\infty) = \sqrt{\left( \frac{\sigma C_{l_\alpha}}{16F} - \frac{\lambda_\infty}{2} \right)^2 + \frac{\sigma C_{l_\alpha}}{8F} \theta r - \left( \frac{\sigma C_{l_\alpha}}{16F} - \frac{\lambda_\infty}{2} \right)}.$$  \hspace{1cm} (12)

Equation 12 can be solved numerically by discretizing the blades into a series of small spanwise elements. It is solved iteratively by first assuming $F = 1$ and then finding $F$ from Equation 4 and recalculating $\lambda$ from Equation 12, up to convergence within a specified percentage. Usually, a few iterations are required to achieve convergence and obtain the inflow solution. BEMT results can provide total thrust coefficient $C_T$ and total power coefficient $C_P$, which allow the calculation of Figure of Merit (FM) by:

$$FM = \frac{C_T^3}{\sqrt{2} C_P}.$$  \hspace{1cm} (13)

In the BEMT model, the airfoil characteristics are included by the 2-D lift-curve-slope of airfoil $C_{l_\alpha}$. A constant $C_{l_\alpha}$ was originally used to analyze the full-scale rotors. However, at low Reynolds number, the lift coefficient curves can be highly nonlinear. For low Reynolds number aerodynamics, under an adverse pressure gradient of sufficient magnitude, the laminar fluid flow tends to separate before becoming turbulent. After separation, the flow structure becomes increasingly irregular. Beyond a certain threshold, it undergoes transition from laminar to turbulent. The turbulent mixing process brings high momentum fluid from the free stream to the near wall region, which can overcome the adverse pressure gradient, causing the flow reattachment and Laminar Separation Bubble (LSB). LSB can increase drag and produce nonlinear lift and moment for the airfoil [8,9]. To increase prediction accuracy, a table look-up was implemented into BEMT model. The lift, drag and moment coefficients for a given airfoil are tabulated as function of local angle of attack and Reynolds number.

The sectional aerodynamic characteristics of Eppler 387 airfoil were obtained from a 2-D Reynolds-averaged Navier-Stokes solver. Standard $k - \epsilon$ turbulent model and $\gamma - \text{Re}_\theta$ transitioned model were employed to simulate fully turbulent flow and transitioned flow, respectively. The calculations were run at the representative Mach number 0.48 and Reynolds number 4,400, for a series of angles of attack. The structured grids (497x112 points) were used to accommodate Eppler 387 airfoil, as shown in Figure 2. The distance from boundary to airfoil was selected as 15c.

Figure 2: 2-D grids on Eppler 387 airfoil.

The Mach number contours from 2-D RANS calculations on the Eppler 387 airfoil are shown in Figure 3 and 4. At angle of attack $\alpha = 0^\circ$, the flow field obtained from $\gamma - \text{Re}_\theta$ transitioned model exhibits a separation vortex which is being shed off the trailing edge of the airfoil. The flow phenomenon fairly agrees with the observations by Mayda et al. in wind turbine airfoil study [10]. They found that, at low angle of attack, the vortex shedding on the Eppler 387 airfoil is generally induced by LSB. However, the LSB is not seen in present study. By contrast, the separation vortex does not appear in the flow field obtained from $k - \epsilon$ turbulent model. At angle of attack $\alpha = 10^\circ$, the separation vortices are clearly observed from the two cases. In contrast to turbulent case, the transitioned case shows a larger vortex structure and earlier separation.

Figure 5 shows the aerodynamic characteristics of Eppler 387 airfoil calculated by 2-D RANS solver. The lift coefficients and lift-to-drag ratio from $\gamma - \text{Re}_\theta$ transitioned model are lower than those from $k - \epsilon$ turbulent model. This is mainly caused by the earlier separation.
Figure 3: Mach number contours at angle of attack $\alpha = 0^\circ$ (Re=4,400, Ma=0.48).

Figure 4: Mach number contours at angle of attack $\alpha = 10^\circ$ (Re=4,400, Ma=0.48).

Figure 5: Aerodynamic characteristics of Eppler 387 airfoil (Re=4,400, Ma=0.48).

Figure 6: Performance predictions of the rotor.
The representative aerodynamic characteristics were then used for BEMT predictions. Figure 6 shows the performance predictions of the proof-of-concept rotor in hover for a series of collective pitch angles. The thrust coefficients and FM values of transitioned case are lower than the counterparts of turbulent case. The maximum value of FM is as low as 0.4, which results from the combination of low Reynolds number and high Mach number. For both cases, the stall behavior of rotor seems to start from the collective pitch of 10°.

3 3-D RANS CALCULATIONS

The proof-of-concept rotor calculations were computed through a 3-D RANS solver, including $k-\varepsilon$ turbulent case and $\gamma - \text{Re}_\theta$ transitioned case. The dual-time stepping scheme was used for the unsteady simulations. The time step was chosen as 0.0001s while the number of sub-iteration was fixed by 20. The calculations were performed for the proof-of-concept rotor with a collective pitch of 10°. The 3-D structured grids on proof-of-concept rotor consisted of near-body part (2,500,000 volumes) and off-body part (2,000,000 volumes), as seen in Figure 7. The radius of the flow domain was set to $12R$. Besides, the distance from inlet to rotor was chosen as $12R$ while the distance from rotor to outlet was increased to $18R$.

Figure 8: Results comparisons between 2-D BEMT and 3-D RANS.

Figure 9: Pressure distributions on the upper surface of rotor blades (units: Pa).
The results of thrust and torque are compared between 2-D BEMT predictions and 3-D RANS calculations, as shown in Figure 8. The results show a clear discrepancy between 2-D predictions and 3-D calculations due to 3-D rotation effect, such as stall modification and root vortices. In addition, the 3-D calculations are not sensitive to the usage of turbulent model or transition model when compared to 2-D predictions. This is further examined by the insight into the physics of pressure distributions. The similar pressure distributions on upper surface of the rotor blades are observed for the two cases, as demonstrated in Figure 9.

For the two cases, the vorticity magnitudes on a slice located 60° behind rotor blades (units: s⁻¹).

For the two cases, the vorticity magnitudes on a slice located 60° behind the rotor blades are shown in Figure 10 shows. The tip vortices are captured successfully by the 3-D RANS calculations. The root vortices are apparent due to the 40% root cut-out, which can not be modeled in present BEMT method.

4 Conclusion

This paper presents 2-D BEMT predictions and 3-D RANS calculations for a proof-of-concept rotor that operates in Martian atmosphere. The proof-of-concept rotor experiences a unique low Reynolds number and high Mach number environment. A BEMT model is implemented to analyze the proof-of-concept rotor. The sectional aerodynamic characteristics used for BEMT is calculated from a 2-D RANS solver. Then, a 3-D RANS solver is employed to investigate the rotor with a collective pitch of 10°. The results show a clear discrepancy between 2-D predictions and 3-D calculations due to 3-D rotation effect, such as stall modification and root vortices. Besides, the 3-D calculations are not sensitive to the selection of turbulent model or transition model when compared to 2-D predictions. The tip vortices are captured successfully by the 3-D RANS calculations. The root vortices occur due to the 40% root cut-out, which can not be modeled in present BEMT method. In the future, the BEMT method is expected to be improved in terms of aerodynamic characteristics correction and root vortex loss. Combining BEMT predictions and RANS calculations, the potential Mars rotor will be designed and assessed.

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References


