Rotary Wing Micro Air Vehicle Endurance

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Abstract

One of the first questions to pilots of rotor based electric driven micro air vehicles is the possible flight time of the system. The answer is not easy sometimes. The possible flight time is influenced by a lot of important details of the micro air vehicle and the flight mission. At the end all components will influence the result. This will start with the battery, continues with the electronic equipment, the mechanical design and ends with the motors and the propellers. Finally the weather condition and the flight mission will influence the maximal possible flight time. This paper starts with a theoretically investigation of the flight performance. The results will be compared with several flight test data from two different rotor based electric driven micro air vehicles. These two vehicles were used during the International Micro Air Vehicle Conferences IMAV 2011 and 2012. At the end this paper will give a battery recommendation for different missions at different weather conditions.

Contents

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1 Symbols

E	=	energy content in [J]
U	=	voltage in [V]
I	=	current in [A]
T	=	time in [s]
m	=	mass in [kg]
d	=	rotor diameter in [m]
g	=	acceleration of gravity = $[9.81 \frac{m}{s^2}]$
h	=	altitude in [m]
v	=	flow velocity in $[\frac{m}{s}]$
ρ	=	density of the air = $[1.25 \frac{kg}{m^3}]$
A	=	sum of the rotor area in $[m^2]$
D	=	specific power of the battery in $[\frac{J}{kg}]$
η	=	efficiency
N	=	number of rotors or battery cells

2 Used test vehicles - quadrocopter "Wanze" and "Ninja"

The two quadrocopter Q-240 "Wanze" and Q-500 "Ninja" were used to validate the theory of this investigation. They are convenient for this comparison because they are similar from the technical point of view. But they have different dimensions. The "Wanze" dimension is half size of the "Ninja" dimension. Both vehicles are an in-house development oft the author. The two vehicles were used during the International Micro Air Vehicle Conferences IMAV 2011 and IMAV 2012, see figure 1.

Figure 1: The quadrocopter "Wanze" and "Ninja" used at IMAV 2011 and IMAV 2012

The air frame based on a sandwich structure made from carbon plates and balsa wood. The wood between the carbon plates is 5 mm thick end grain. The result is a very stiff and light design. The complete electronic closed loop control will be performed by a microprocessor ATMEGA328 on a printed circuit board with the dimension of 20 x 30 mm. The three gyroscope sensors ADXRS610 measure the rotation speed of the vehicle. The three acceleration components will be measured by the sensors ADXL322. All sensors are implemented in a 17 mm red cube, see figure 1 . This cube was used in different other projects, see references [4], [3] and [5]. The software is an in-house development as well. The basis is the Arduino platform, the size of the code is 8 Kbytes about. Part of the code is the closed loop control including the rotary matrix. The closed loop frequency is 400 Hz. The RC receiver is a single satellite from a standard Spectrum receiver. The receiver is connected at the serial port of the controller board. The standard electronic speed controllers receive typical pulse wide modulation information 1.1 ms to 1.9 ms from the controller. For the "Wanze" AirAce 3-blade left/right propellers were used, the diameter is 100 mm. The motors are Hacker A10/12S powered by electronic speed controller Hype 6 Ampere. The basic weight of the quadrocopter is 119 gram. The dimensions of the "Ninja" are bigger than the "Wanze" dimensions. Scaling factor is two. The "Ninja" is using GWS 8x4 3-blade left/right propellers. The diameter is 200 mm. The propellers are powered by Axi motors 2808/34 and electronic speed controller Hype 10 Ampere. The basic weight of the "Ninja" is 352 gram. Both quadrocopter were used to validate the theory described in the next chapters.

3 Battery - the source of the energy

Lithium polimer batteries (Li-Poly) were used in the area of model aircraft presently. The main parameters are mass and specific power of the battery. The energy content can be calculated from the voltage, the current and the discharge time, see equation (1).

$$E = U \cdot I \cdot T \tag{1}$$

To have an impression of the energy content of a lithium polimer batteries we can do a thought experiment. The energy content from equation (1) can be used to lift the battery only. The idea is to convert this electrical energy to mechanical energy. This energy can be calculated from equation (2). The energy will be calculated from the mass, the acceleration of gravity and the lift altitude.

$$E = m \cdot g \cdot h \tag{2}$$

In the thought experiment we suppose the electrical energy will be transformed to the mechanical energy completely. We assume there is no loss of energy and the mass of the motor and the wire is negligible. The set up maybe looks like figure 2.

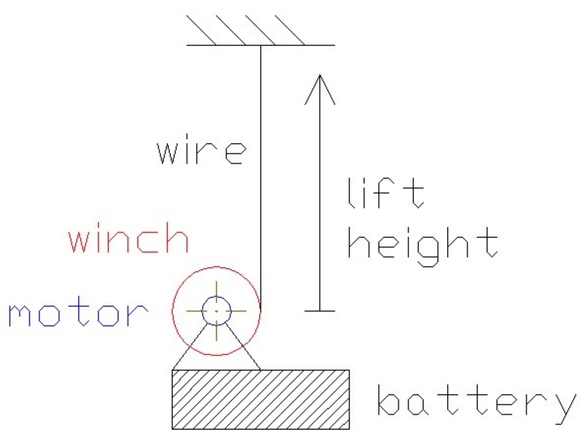

Figure 2: The set up of the battery thought experiment

The electrical energy from the battery will be transformed to lift energy completely. The energy from equation (1) and equation (2) is identical, so the lifting altitude can be calculated from equation (3).

$$h = \frac{U \cdot I \cdot T}{m \cdot g} \tag{3}$$

Example: Lithium polimer batteries with 3 cells and a capacity of 1 500 mAh has a mass of 142 gram. He is able to lift his own weight in the thought experiment up to 43 kilometer see equation (4). In the reality this value will be smaller because of losses in all components. But the value of 43 kilometer is very high, he shows how much energy is installed in a lithium polimer battery.

$$h = \frac{11.1 \cdot 1.5 \cdot 3600}{0.142 \cdot 9.81} \text{ meter} \approx 43 \text{ kilometer.} \tag{4}$$

The fact that this value is not dependent from the size of the battery is remarkable. A heavier battery will have more electrical energy. The quotient of energy and mass is similar for one type of battery. It is possible to calculate a specific power of the battery. This quotient shows how much energy is installed in one kilogram of the battery. In table 1 several lithium polimer batteries are listed. They are sorted by the number of the cells, the mass and the capacity.

number of cells	voltage/[V]	mass/[g]	capacity/[mAh]	energiy density/[J/kg]
2	7,4	28	350	333000
2	7,4	20	350	466200
2	7,4	27	450	444000
2	7,4	30	500	444000
2	7,4	43	740	458456
2	7,4	41	800	519805
2	7,4	68	1200	470118
3	11,1	41	450	438585
3	11,1	78	910	466200
3	11,1	142	1500	422113
3	11,1	165	1800	435927
3	11,1	329	3600	437252
			average value=	444638

Table 1: Overview on several lithium polimer batteries

For further calculations we suppose an average specific power of the battery D of 444 kJ/kg. Of course always there will be examples with a higher or smaller specific power, but the value 444 KJ/Kg is a good approximate value. With this value we can estimate the mass of several lithium polimer batteries, see equation (5).

$$m_{battery} = \frac{E}{D} \tag{5}$$

A lithium polimer battery with 3 cells and a capacity of 10 000 mAh will have a mass of 0.9 kg for example. This calculation we need later, when we select an optimal battery for several missions and quadrocopter applications.

4 Propeller - there the thrust is coming from

A good overview on the aerodynamic of propellers is given in the chapter 4 of the reference [1]. The air above the propeller goes to the propeller and creates a jet below the propeller, see figure 3.

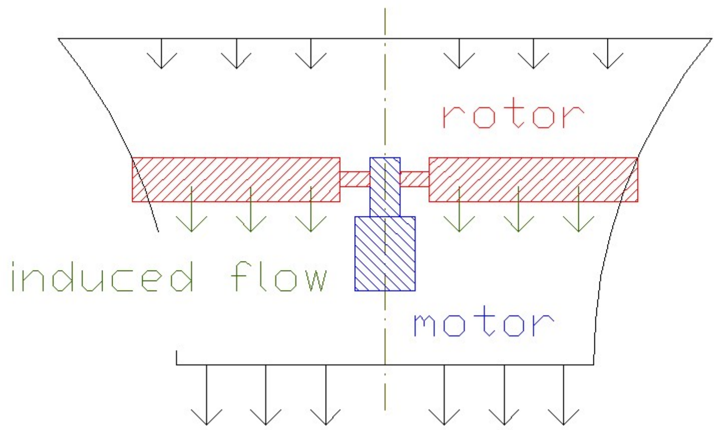

Figure 3: The induced flow velocity near to the propeller plane

In the propeller plane there is an induced velocity. This velocity can be calculated from equation (6).

$$v = \sqrt{\frac{m \cdot g}{2 \cdot \rho \cdot A}} \tag{6}$$

The induced velocity is high, when the air vehicle has a high weight and a small rotor area. To calculate the installed thrust power we have to multiply the induced velocity by the thrust. As result we have an equation to calculate the necessary power for the hover flight, see equation (7).

$$P = \frac{(m \cdot g)^{1.5}}{\sqrt{2 \cdot \rho \cdot A}} \tag{7}$$

The power is dependent from density of the air, the acceleration of gravity the mass of the vehicle and the rotor area. A heavy quadrocopter with small propellers needs more power. To fly with a small amount of power it is necessary to use big propellers on a quadrocopter with a low mass. It is possible to decrease the amount of energy by using the ground effect. The level of the induced velocity is limited because of blockage and this limits the value of power.

5 Flight time - this time the air vehicle can stay in the air

The flight time can be calculated by the energy content and the necessary power of the air vehicle, see equation (8).

$$T = \frac{E}{P} \tag{8}$$

No we use equation (5) and (7) in the equation (8). And we assume the flight mass is the sum of the empty weight of the air vehicle and the mass of the battery. Now we have an equation for the flight time, see equation (9).

$$T = \frac{E \cdot \sqrt{2 \cdot \rho \cdot A}}{(g \cdot (m_{empty} + m_{battery}))^{1.5}} = \frac{E \cdot \sqrt{2 \cdot \rho \cdot A}}{(g \cdot (m_{empty} + \frac{E}{D}))^{1.5}} = \frac{m_{battery} \cdot D \cdot \sqrt{2 \cdot \rho \cdot A}}{(g \cdot (m_{empty} + m_{battery}))^{1.5}} \tag{9}$$

To select a battery for a flight mission we can use equation (9) for the calculation of the possible flight time. A high value for the energy content and a large propeller area will produce long flight times. More difficult is the situation for the mass of the battery. This value is included in the numerator and the denominator of the equation (9). In the next chapter we will see there is a optimal battery mass to reach a maximum for the flight time in equation (9).

Two characteristic curves were shown in figure 6 for the quadrocopter "Wanze" and "Ninja". Important is the fact the real flight time will be smaller then the calculation from equation (9). This theoretical value we have to multiply be an efficiency of the system. This effect is caused by losses in the battery, electronic, motors and propellers. We add all this losses in one efficiency factor for the complete system, see equation (10).

$$T_{effective} = \eta \cdot T \tag{10}$$

The efficiency factor we can determine in flight tests. The size of the battery was varied, see figure 4 and figure 5 .

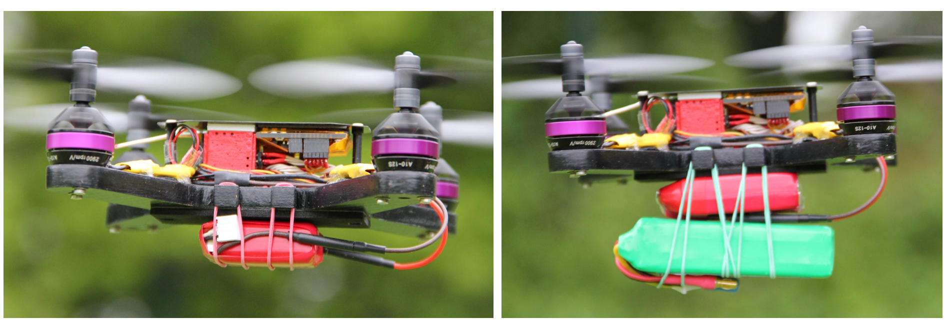

Figure 4: Comparison of different batteries for the quadrocopter "Wanze"

We compare the theoretical flight time versus the real flight time. The quotient is the efficiency factor. This efficiency is dependent on the components of the air vehicle. To simplify the handling of the equation (9) we can assume a voltage of one cell of the lithium polimer battery of 3.7 Volt. The density of the air will have a value of 1.25 $\frac{kg}{m^3}$. The acceleration of gravity is 9.81 $\frac{m}{s^2}$. These information are implemented in the new factor 0.32, see equation (11). With this modification we will have an practical equation to estimate the possible flight time in minutes.

$$\frac{T_{effective}}{[minutes]} = \eta \cdot 0.32 \cdot \frac{\sqrt{N_{rotors}} \cdot \frac{d_{rotor}}{[mm]} \cdot N_{cells} \cdot \frac{capacity}{[mAh]}}{(\frac{m_{empty}}{[gram]} + \frac{m_{battery}}{[gram]})^{1.5}} \tag{11}$$

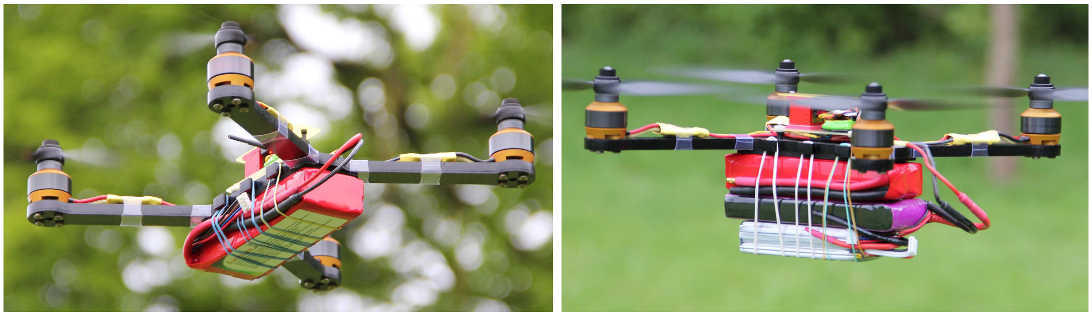

Figure 5: Comparison of different batteries for the quadrocopter "Ninja"

In figure 6. comparison between the calculated and the tested flight times are shown. The results fits well if we consider an efficiency factor for the quadrocopter "Wanze" of 0.19. For the quadrocopter "Ninja" this value is 0.35. The lower value for the "Wanze" based on the smaller propellers. The small dimension of the propeller will have addition aerodynamic losses. Since the same electronic components were used in both quadrocopter the constant losses of these components we decrease more the global efficiency of the smaller quadrocopter "Wanze".

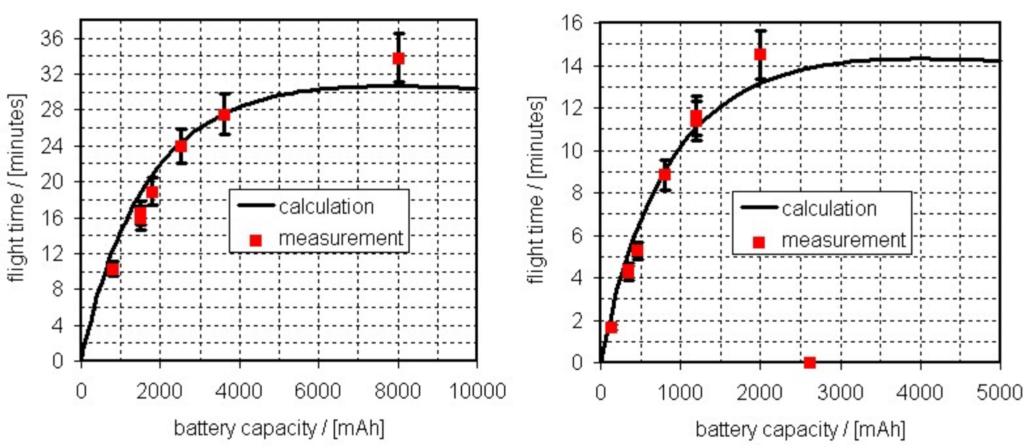

Figure 6: Comparison calculated versus measured flight time of the "Wanze" and "Ninja"

The "Wanze" was not able to lift the battery with the capacity of 2600 mAh. On the left hand side of figure 6 the flight time was quoted with 0 minutes. For practical applications is has to be ensure the air vehicle is able to lift lift the battery. This is an additional limitation in the selection of the batteries for a mission. The comparison of the flight times for the "Wanze" and the "Ninja" shows the measured flight times for higher capacities of the battery is higher than the calculated values. Reason for this is the increase of efficiency for higher thrust levels. The aerodynamic condition increase with the increase of flow velocity for higher lift levels, based on a higher Reynolds-Number.

6 Optimal battery mass - more flight time is not possible

In figure 6 we see there is a optimal flight time. If we increase the size of the battery there is no longer flight time possible. This optimum can be found by the investigation the quotient (12) in equation (9).

$$\frac{m_{battery}}{(m_{empty} + m_{battery})^{1.5}} \tag{12}$$

The flight time has the maximal value when the mass of the battery is twice as high then the mass of the empty air vehicle. This is an easy result, see equation (13).

$$m_{battery} = 2 \cdot m_{empty} \tag{13}$$

This result is visible in figure 6. The result is not depended from the size of the air vehicle. It is not important which type of battery is used. Different type for example nickel cadmium batteries will have different flight times but the optimum is still reached when the battery mass is twice as high then the mass of the empty air vehicle. For the quadrocopter "Wanze" the value for the maximal flight time would be 3967 mAh. But this set up was not possible. The "Wanze" was not able to lift this battery. For the "Ninja" the optimal value was 7822 mAh. This value was tested and validated in the flight tests, see figure 6 .

7 Selection of the battery - the optimal battery for different flight missions

For the selection of the battery several point have to be investigated. First the number of cells will be adjusted. For this point the rotor and motor specification is important. A good selection for the quadrocopter "Wanze" was 2 cells and for the "Ninja" 3 cells. This delivers enough thrust and is a good combination for the motor and the rotor speed. To optimise the battery for the flight mission it is necessary to distinguish between different flight missions. Typical missions A, B, C and D are possible:

7.1 Mission A: Hobby flights without special needs

To enjoy in flying with the air vehicle a small battery is a good selection. This limits the costs for the battery and decreases the crash mass of the vehicle. The mechanical loading of the frame will not have a high value in a crash situation. The battery mass should be 50 % of the mass of the empty air vehicle. This is a good estimation for the first flights. The quadrocopter "Wanze" uses for this mission a battery with 2 cells and a capacity of 800 mAh. The possible flight time is 9 minutes. The quadrocopter "Ninja" will use a 3 cells battery with the capacity of 1500 mAh battery and reaches a flight time up to 16 minutes.

7.2 Mission B: Pylon races or speed or climb competitions

There we need power and a low mass of the air vehicle. Maybe a different number of cells is profitable for this mission. But at the end the best solution is the smallest battery, who is able to fulfil the mission. Maybe the necessary flight time is 1 to 2 minutes only. Important is the fact, that the needed power is higher for this mission than for the calculated needed power for hovering flight. Experimental results expect an increase of power for this mission by 40 % see references [2]. The battery capacity has to be enlarge by this value.

7.3 Mission C: Long range flights - fly the maximum possible time

For this mission the mass of the battery mass should be twice as high then the mass of the empty air vehicle. With this battery set up we reach the maximum flight time. There is no need to use bigger batteries, the flight time will be smaller. If we select a small battery the flight time will shorter. But if we use a battery with the same mass then the empty air vehicle the possible flight time is 92 % of maximal possible flight time. Maybe there is no need to reach the last 8 % by doubling the battery mass and cost. This is related to more noise and a higher crash mass of the air vehicle.

7.4 Mission D: Stable flights - steady flight under windy weather conditions

There we need a higer mass of the quadrocopter. With a high mass of the air vehicle we reach a high induced flow velocity near to the rotor plane, see equation (6). If there is an additional gust based from the weather turbulence, then will be a change of the incoming flow at the propeller blade. The local wind spee and the angle of attack can be different from the reference flow condition. This will increase or descries the local lift at the propeller blade element. This will end up with additional oscillations of the air vehicle. We have to avoid this for flights with cameras an other payload. Often there is the need for a stable flight. The disturbance of the reference flow condition is bigger for smaller induced flow velocities in the propeller plane. If there is a high level of induced flow velocity available, then an additional gust will have less influence on this reference flow condition, see figure 7.

The flow condition on a propeller blade element is shown in the upper part of the figure 7. The incoming flow on the blade is the sum of the induced flow and the peripheral speed of the propeller. If there is an additional gust, see lower part of the figure 7, then the incoming flow on the propeller will be different. The change in the angle of attack is bigger for lower velocities, see left hand side of the lower part of figure 7. If there is a high velocity of the induced flow and the peripheral speed of the propeller then a turbulence gust is not able to disturb this flow condition. The change in the angle of attack is smaller then for the condition with lower speed. Since the angle of attack is bigger, the flow conditions are less stable. This will induce more oscillations of the vehicle and there is the need to have more compensation from the closed loop control of the air vehicle.

The higher flow velocities we will achieve by a high mass of the flight vehicle. There is no mandatory need to increase this high mass with the battery only. It can be used an additional mass or a heavier structure mass of the airframe to. If the quadrocopter is able to lift this additional mass, then this is a good set up for this mission. It can be used a mass higher than for mission C but the flight time will be smaller. At the end the flight will be more stable.

For all Missions is important to know, there are more flight limitations. For very small batteries in mission B it is necessary to check if the batteries are capable to deliver the electrical current for the mission. For heavy batteries the first check should be the comparison of the maximum available lift from air vehicle. For all mission buffers in the capacity of the batteries should be included.

8 Outlook

There will be more powerful batteries in the future. More specific power of the battery will be available. This will increase the maximal possible flight time. To reach long flight times today

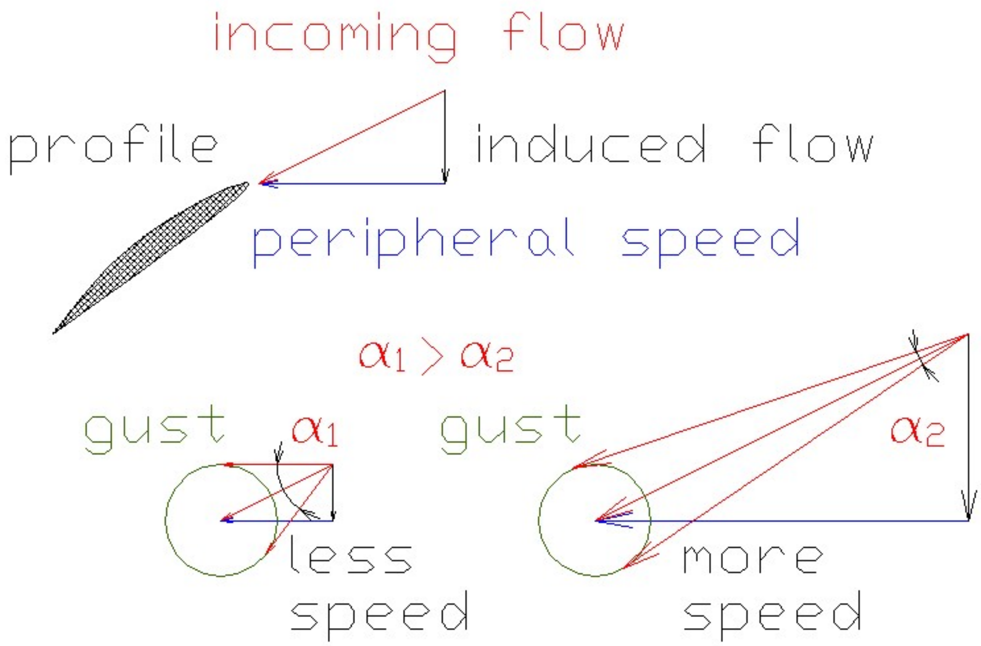

Figure 7: Flow conditions on a rotor blade element for low and higher flow speeds

there is the need to select all components of the air vehicle carefully. Flights in a low level turbulence condition will increase the flight time. Additional the ground effect can increase the maximal possible flight time. But for this environments there are no interesting missions available unfortunately. An other topic is to use more batteries for longer missions. They can be dropped stepwise after discharging. To estimate the flight time of rotor based electric driven micro air vehicles there is the need to investigate a lot of details of the vehicle. The other solution is to perform flight experiments. The best solution is to have the right balance between theoretical investigations and experimental flight tests.

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