OPTIMAL CONTROL OF FLAPPING WING WITH TAKING INTO ACCOUNT NONSTEADY EFFECTS

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ABSTRACT

Considered in this paper is the task of the flapping wing trajectory optimization for the minimization of the wing consumed energy associated with the induced power losses. The aircraft with such a wing assumes to move in the horizontal direction with the constant velocity. The flapping wing moves up-and-down with the constant velocity. Nonsteady vortex wake influence is analyzed. Comparison of the efficiency is performed for the steady case, as well as for the one with the sinusoidal and optimal control for the flapping wing. Also, comparison with another type of the thrust creation (propeller) is performed.

1 INTRODUCTION

The scientists are motivated to consider different problems in the field by the interest produced by the flapping wing and the aircrafts with such wings. For example, Jones et al. [1] analyzed the wake structure behind plunging airfoils. They compared the picture obtained with the aid of the numerical method for the inviscid incompressible flow and the experimental results and shown that the plunging airfoil can produce drag, zero drag or thrust depending on the motion parameters (reduced frequency and plunging amplitude). The trust coefficient was obtained and it was note that for the too small or large reduced frequencies the numerical and the experimental results no longer coincide. It is because of the significant viscous influence in the first case and the flow separation in the second one. Tuncer and Platzer [2] compared the numerical results for the plunging airfoil obtained with the help of the different methods: for the inviscid incompressible flow, for the hybrid method with the Navier-Stokes equations solving in the boundary layer and for the viscous flow. It was shown that these methods give the close results (Re ~10^6). They also investigated the questions of the plunging airfoil efficiency and of the two airfoils in tandem interaction and their efficiency. Tuncer and Kay [3] solved the optimization task for the thrust and efficiency maximization for the airfoil in a combined plunge and pitch with the plunge and pitch amplitudes and phase shift between them as optimization parameters. Then they and Jones and Platzer [4] solved similar problem for the two airfoils in a biplane configuration. Nagai and Hayase [5] investigated numerically and experimentally aerodynamic characteristics of the insect wing in the forward flight and its efficiency. Bermang and Wang [6] considered the case of the hovering insect flight and have found optimal wing kinematic which minimize power consumption. Wang [7] has sought the simplest efficient flapping motions with the aid of the model of the quasi-steady forces, and has made the comparison with the steady forward flight.

But up to now, in our opinion there are no enough studies concerned the question about optimal kinematic of the flapping wing motion. So this work deals with this question; analytical and numerical investigations were made for the flapping wing efficiency determination for a typical case. The results obtained were compared with the “traditional” type of the thrust generation (propeller).

2 MODEL

Flight of the aircraft in the horizontal direction with the constant speed at the steady altitude is considered. The model of the aircraft investigated is shown schematically in fig.1; it has the fixed wing for the lift production and the flapping wing (or wings) only for the thrust creation (as in [8]). The flapping wing creates also the lift at every moment of the motion but since the lift is positive during downstroke and negative during upstroke the period-average lift is zero. Besides, the interaction between the fixed wing and the flapping one was ignored.

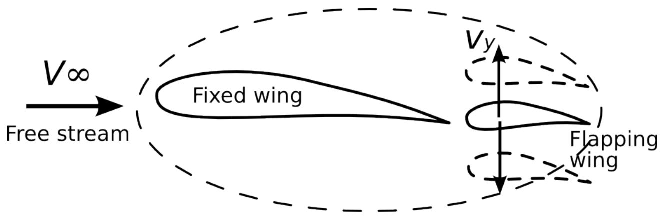

Fig.1 Model of the aitcraft

It is assumed that the wing is perfectly rigid (do not change its form under load) and its mass and moment of inertia are zero (i.e., wing speed and its orientation in space can change instantly). It has shown early [9] for the case without taking the nonsteady effect into consideration that the flapping wing should moves in a straight line with the constant velocity for the consumed power minimization. So in this study the flapping wing performs the plunging motion moving up and down with the constant velocity v_y. It can also perform the pitching motion. The oscillation amplitude and frequency are considered so that the vortex wake remains nearly plane. The diagram of the forces acting on the flapping wing is presented in fig. 2.

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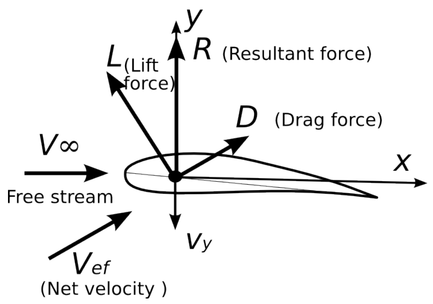

Fig. 2 Diagram of the forces on the flapping wing

It was shown in [10] that there exists a region of the parameters of the flapping wing motion (reduced frequency and amplitude) where the Kutta condition is valid. So, assume that our wing parameters correspond to this region. This means that the flow is coming off the trailing edge of the wing without separation.

3 VORTEX WAKE INFLUENCE EVALUATION

The problem in a simplified statement (model task) has been considered to understand the main features of the processes taken place.

The wing is modeled with the bound vortex and two free vortices coming off the wing tips and closing to “rings” when the vertical velocity component is reversed (Fig.3). Assume that the circulation distribution is constant along the wing.

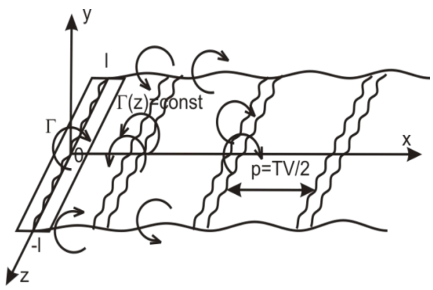

Fig.3 Simplified model for nonstationarity influence investigation

It is well known [11] that the induced velocity V_i from the vortex section at a certain point of observation is given by the formula

$$V_i = \frac{\Gamma}{4\pi h}\left(\cos\theta + \cos\varphi\right)$$

where Γ is the circulation, h is the length of perpendicular between the observation point and the line containing the vortex section, θ and φ are the angles between the lines from the observation points to the ends of the vortex section and the line of the vortex section (see Fig.4).

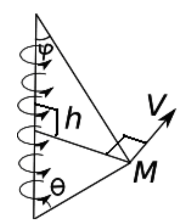

Fig. 4 Vortex section

The analytical formula was found for the induced velocity created at a certain wing point $M(z)$ at a certain moment of wing motion as the sum of the velocities generated by all the vortex sections:

$$V_i^{sum}(x,z) = \frac{\Gamma}{2\pi}\sum_{i=1}^{\infty}\frac{(-1)^i}{x+(i-1)p} \times \left(\frac{z}{\sqrt{(x+(i-1)p)^2+z^2}} + \frac{L-z}{\sqrt{(x+(i-1)p)^2+(L-z)^2}}\right) -$$
$$- \frac{\Gamma}{4\pi p}\frac{x^2}{z}\frac{1}{\sqrt{x^2+z^2}} - \frac{\Gamma}{4\pi p}\frac{x^2}{L-z}\frac{1}{\sqrt{x^2+(L-z)^2}} +$$
$$+ \frac{\Gamma}{4\pi z}\sum_{i=1}^{\infty}(-1)^i \times \left(\frac{x+(i-1)p}{\sqrt{(x+(i-1)p)^2+z^2}} + \frac{x+ip}{\sqrt{(x+ip)^2+z^2}}\right) +$$
$$+ \frac{\Gamma}{4\pi(L-z)}\sum_{i=1}^{\infty}(-1)^i \times \left(\frac{x+(i-1)p}{\sqrt{(x+(i-1)p)^2+(L-z)^2}} + \frac{x+ip}{\sqrt{(x+ip)^2+(L-z)^2}}\right).$$

Since the above expression is too complicated for the analytical investigation, the numerical analysis was conducted for the detection of the main features.

As the value of the circulation at the wing tips must always be equal to zero [11], let us specify the circulation function as:

$$\Gamma=\Gamma_0,\ \varepsilon<z<L-\varepsilon,$$
$$\Gamma=0,\ 0<z<\varepsilon,\ L-\varepsilon<z<L,$$

where ε is a certain parameter ε«L, L – wing span. The following parameters were taken for calculation: $L=2m$; $\Gamma_0=1m^2/s$; $\varepsilon=0.01L$; $b=0.1L$; p – over the range from $0.5L$ to $10L$., where b is the wing chord.

Such are the main results. The induced force value averaged for a time period and for the wing span was obtained. The nearest to the wing vortices was found to give the main contribution. The induced velocities of the vortex wake generate on the wing alternate in direction forces, the first of them being positive (drag).

The comparison with the stationary case was made, the wing being modeled with the bound vortex and two infinite free vortices (Fig.5).

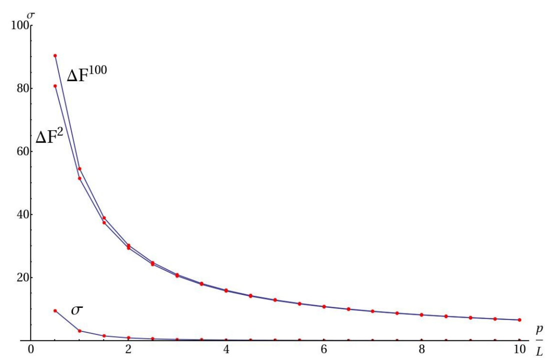

Fig.5 Comparison with the steady case

Here, ΔF^{100} is the difference between the force in the steady case and the force from the first hundred vortex circular elements, ΔF^2 is the difference between the force in the steady case and the force from the first two vortex circular elements and σ is the difference between ΔF^{100} and ΔF^{2}. As shown in the figure the neglecting of all the vortices, starting with the third one, gives the error of the several percent if the value of the parameter p/L is not too small. The force dependence on the wing position during the period is shown in fig. 6.

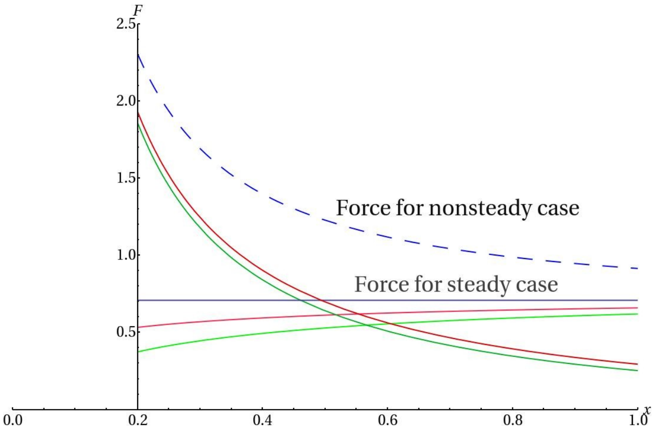

Fig.6 Plot of force vs. the wing position during the period

The horizontal line corresponds to the force in the steady case; dashed line corresponds to the nonsteady force. The light-green line and the green one are the forces from the first perpendicular and parallel to the wing vortices accordingly.
The light-red line and the red one are the forces from the two nearest to the wing vortices. One can see that the significant extra drag force appears at the beginning of the period and then decreases fast, and during the rest of the time the movement is close to the stationary case.

4 OPTIMIZATION TASK

It is obvious that the instantaneous changing of the circulation value may not correspond to the optimal case, so for simplicity let's assume that the circulation distribution along the wing span is constant but suppose, that the circulation time dependence is a certain periodical function Γ(*t*), that is due to the wing pitching motion (Fig.7).

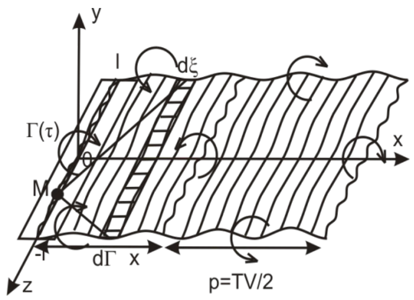

Fig.7 Vortex sheet investigated

At every moment of the motion, the horseshoe vortex with the circulation $d\Gamma = \frac{1}{V_\infty}\frac{\partial \Gamma}{\partial \tau}d\xi$, where $\tau = t - x/V_\infty$, leaves the wing. The span-averaged induced velocity generated with the vortex wake of length of 2*p* is:

$$v_i = \frac{1}{8\rho l V_\infty}\int_0^{2p}\Gamma'_\tau\int_{-l+\varepsilon}^{l-\varepsilon}\left[\frac{1}{x}\left(\frac{l-z}{\sqrt{x^2+(l-z)^2}}+\frac{l+z}{\sqrt{x^2+(l+z)^2}}\right)+\frac{1}{l-z}\left(-\frac{x}{\sqrt{x^2+(l-z)^2}}+\frac{2p}{\sqrt{(2p)^2+(l-z)^2}}\right)+\frac{1}{l+z}\left(-\frac{x}{\sqrt{x^2+(l+z)^2}}+\frac{2p}{\sqrt{(2p)^2+(l+z)^2}}\right)\right]dx.$$

The two vortices coming of the wing tips should be also taking into account. They induce the velocity:

$$u_i = \frac{\Gamma}{4\pi l}\int_{-l+\varepsilon}^{l-\varepsilon}\frac{dz}{l-z}.$$

The induced force is $F=\rho(v_i+u_i)\Gamma$, and the required power for this force generation is $W_i=FV_\infty$. The problem is to minimize the period-average induced power W_i, that is:

(1) $$I_0 = \frac{1}{T}\int_0^T W_i dt = \frac{\rho V_\infty}{T}\int_0^T \Gamma(v_i+u_i)dt,$$

provided that the power spent to the oscillatory motion is:

(2) $$\rho V_\infty V_y\left(\int_0^{\frac{T}{2}}\Gamma\,dt - \int_{\frac{T}{2}}^{T}\Gamma\,dt\right) = \rho V_\infty V_y B = const.$$

As we assume that the flapping wing produces no lift in average, then the following condition must be taken into account:

$$\int_0^T \Gamma\,dt = 0.$$

Let's expand the periodical circulation function in the Fourier series and determine the zero moment of time so that the sine function will only present in the expression:

(3) $$\Gamma(t) = \sum_{n=1}^{\infty} a_n \sin\left(\frac{2\pi nt}{T}\right).$$

Assume that the interchange the operations of summation and integration can be done. By substituting expression (3) into (1), (2) and carrying out some transformations, one can obtain:

$$I_0 = \frac{\rho V_\infty \ln\frac{2l}{\varepsilon}}{8\pi l}\sum_{n=1}^{\infty}a_n^2 + \frac{\rho Int V_\infty}{16lp}\sum_{n=1}^{\infty}na_n^2,$$

$$Int = \int_0^{2p}\sin\left(\frac{\pi nx}{p}\right)\Phi(x,\varepsilon)dx,$$

where Φ is the expression in the square brackets from formula for v_i after integration with respect to z.

Then, the optimization task can be reformulated: it is required to minimize the function

$$I = \frac{\ln \frac{2l}{\varepsilon}}{\pi} \sum_{n=1}^{\infty} a_n^2 + \frac{Int}{2p} \sum_{n=1}^{\infty} n a_n^2 ,$$

under condition

$$\sum_{\tilde{n}=1}^{\infty} \frac{a_{\tilde{n}}}{\tilde{n}} = \frac{\pi B}{2T} ,$$

where tilde over n means that odd summand only appear in the sum, since the rest is nulled after the integration. Lagrange function for this task is:

$$L = \frac{\ln(2l/\varepsilon)}{\pi} \sum_{n=1}^{\infty} a_n^2 + \frac{Int}{2p} \sum_{n=1}^{\infty} n a_n^2 + \lambda \left(\sum_{\tilde{n}=1}^{\infty} \frac{a_{\tilde{n}}}{\tilde{n}} - C_0 \frac{\pi}{2\rho v_y V_\infty} \right)$$

where λ is the Lagrange coefficient, C_0 is the constant from condition (2).
Optimum conditions are as:

$$\frac{\partial L}{\partial a_i} = 0, i \in [1, \infty), \frac{\partial L}{\partial \lambda} = 0 .$$

Really optimal solution can be obtained only if all the harmonics of oscillations are taken into account. But this task is very difficult to solve and to analyze. Also, it is well known that usually the amplitudes of higher harmonics are rather small in comparison with the first ones. So, it was decided to analyze the optimization problem only for a series of the first harmonics.

The solution for the optimization task was found for a set of values of n for a set of values of C_0 (the constant *const* from condition (2)). For example, the circulation function of optimal control is given in fig. 8,9 for 1, 2, 3, 4, 5 and 15 harmonics under conditions C_0=30 W.

For n more or equal 3 the graphics are rather close to each other (as it was assumed above), so we may content with the first several harmonics.

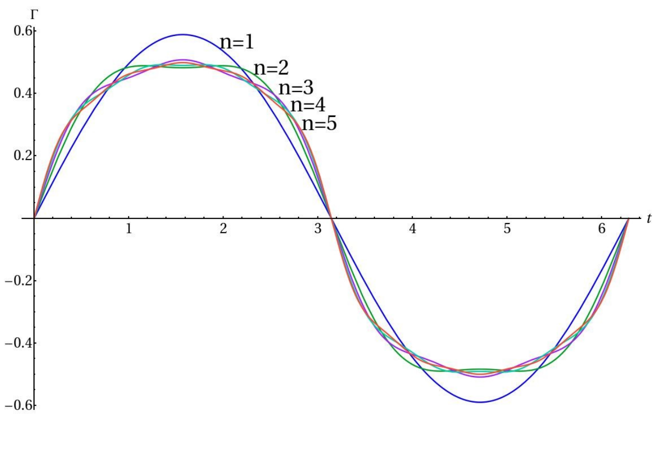

Fig.8 Function of optimal control obtained; n=1,2,3,4,5

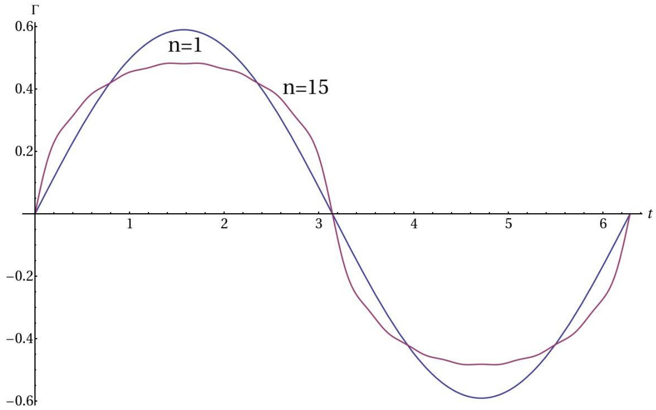

Fig.9 Function of optimal control obtained; n=1,15

C_0,W	η_1	η_2	η_3	η_4	η_5
1	0.99088	0.99129	0.99138	0.9914	0.9914
10	0.89971	0.90423	0.90522	0.9056	0.9058
20	0.80855	0.81717	0.81905	0.8198	0.8201
30	0.71739	0.73011	0.73289	0.7339	0.7344
40	0.62622	0.64306	0.64673	0.6481	0.6488
50	0.53506	0.55600	0.56056	0.5623	0.5631

Tab.1 The numerical comparison the efficiencies for optimal and sinusoidal control p/l=1

C_0,W	η_1	η_2	η_3	η_4	η_5
1	0.99396	0.99427	0.99434	0.99436	0.9944
10	0.93359	0.93697	0.93773	0.93802	0.9381
20	0.87323	0.87967	0.88113	0.88169	0.8819
30	0.81286	0.82238	0.82453	0.82535	0.8257
40	0.75249	0.76508	0.76793	0.76901	0.7695
50	0.69213	0.70779	0.71133	0.71267	0.7133

Tab.2 The numerical comparison the efficiencies for optimal and sinusoidal control p/l=2

5 Efficiency determination and comparisons

It is interesting to compare the gain with the optimal control of the wing in comparison with more often used and more easily implemented sinusoidal movement law (number of harmonics is 1). Let's determinate the efficiency as the rate of the period-averaged useful power (i.e., the one suitable to wing thrust) to the mean full power (i.e., the one spent to the oscillatory motion).

$$W_T = \frac{1}{T} \int_0^T \rho \Gamma V_\infty (v_y + v_i + u_i) dt$$

$$W = \frac{1}{T} \int_0^T \rho \Gamma V_\infty v_y dt$$

$$\eta = \frac{W_T}{W} = 1 - \frac{\int_0^T \Gamma (v_i + u_i) dt}{v_y \left(\int_0^{\frac{T}{2}} \Gamma \, dt - \int_{\frac{T}{2}}^T \Gamma \, dt \right)}$$

And after some transformations the formulas will take the form:

$$\eta = 1 - \frac{\rho Int V_\infty}{16 pl C_0} \sum_{n=1}^{\infty} n a_n^2 - \frac{\rho V_\infty \ln(2l/\varepsilon)}{8\pi l C_0} \sum_{n=1}^{\infty} a_n^2$$

The numerical investigation was made for a set of parameters: l=1m, p=2l and p=l, v_y=2m/s. The results are given in Tab.1 and Tab.2 for a set of values C_0; η_2 – the efficiency for the optimal control case when the optimization task is solved for n=2 (the circulation function includes two harmonics); η_3, η_4, η_5 are defined similarly when n=3 and n=4; n=5; η_1 is the efficiency for the sinusoidal control.

It is obvious that the increase of the number of harmonics more than two practically do not produce any changes in the efficiency advantages. The optimal control utilization gives a little gain and it is the smaller the longer period of the motion.

The comparison of the efficiency for the steady case with the sinusoidal control one and the optimal control (n=5) one was made. The plot of $\eta(T)$, where T is the wing thrust, is presented in fig. 10.

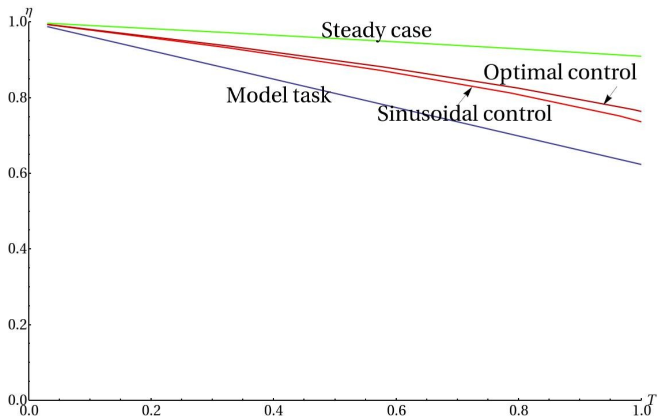

Fig.10 Efficiency versus the wing thrust for the steady and the nonsteady cases

6 Comparison with the ideal propeller

Assume the propeller will be taken to be the flapping wing equivalent if they are of the same swept area. Then, the equivalent propeller diameter is

$$D = 4\sqrt{\frac{pl}{\pi}\frac{v_y}{V_\infty}} \tag{4}$$

The coefficient of the efficiency of the ideal propeller is defined as [10]:

$$\eta = \frac{1}{1 + \frac{2T}{\rho V^2 \pi D^2}}$$

The coefficient of the efficiency of the ideal propeller is much more than the efficiency of the flapping wing, even for the steady case (fig.11).

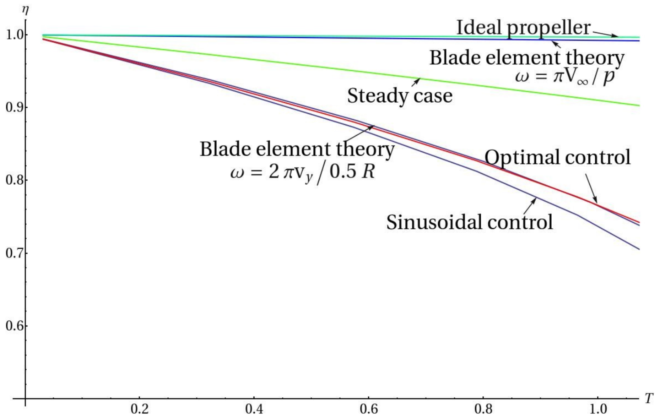

Fig.11 Efficiency comparison for p/l=2

7 Comparison with the propeller blade element theory

The quasilinear statement of the problem is used, i.e. the diameter of the stream from the propeller remains practically constant [12]. Assume the circulation of the bound vortex along the blade is constant. Then, the vortex sheet is the vortices coming of the blade tip and the hub. Let us define the angular velocity ω of the equivalent propeller (4) so that the rotary velocity for the characteristic section at 0.5R to be equal to v_y:

$$\omega = \frac{2\pi v_y}{0.5R}. \tag{5}$$

The efficiency for such propeller is

$$\eta = \frac{C_T \bar{V}}{m_K};$$

$$C_T = \int_{r_0}^{1} 8\bar{\Gamma}\bar{U}_1 d\bar{r}, \quad m_K = \int_{r_0}^{1} 8\bar{\Gamma}\bar{V}_1 \bar{r} d\bar{r}$$

are the thrust and power coefficients of the propeller, $\bar{V}$ is the free stream relative velocity, $\bar{U}_1 = \bar{r} - \bar{u}_1$, $\bar{V}_1 = \bar{V} + \bar{v}_1$ are the relative velocity components of the real stream, $\bar{v}_1 = -\frac{\bar{V}}{2} + \sqrt{\left(\frac{\bar{V}}{2}\right)^2 + \bar{\Gamma}(1-\bar{\Gamma})}$ and $\bar{u}_1 = \frac{\bar{\Gamma}}{\bar{r}}$ are the axial and the peripheral components of the induced velocity. The velocities marked by bar are related to the blades tips rate ωR, $\bar{\Gamma} = \frac{\Gamma}{4\pi\omega R^2}$. The dependency of $\eta(T)$ was obtained for the same parameters that in the flapping wing case (fig.11). It is practically the same the efficiency of flapping wing (in the case of p/l=2). The dependency for the p/l=1 is presented in fig.12.

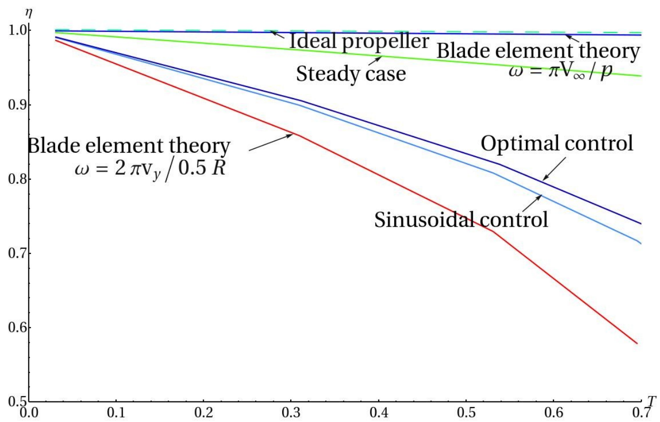

Fig.12 Efficiency comparison for p/l=1

As shown in the figure the efficiency of the propeller is less than for the flapping wing one.

Also it should be mentioned that the results of comparison strongly depend on the method of ω determination (see (5)). If ω is defined so that the distances which the vortex wake passes during the one swing and during the one propeller period are equal:

$$\omega = \frac{\pi V_\infty}{p}$$

the propeller efficiency is appeared more higher for both parameters p/l (fig. 11,12). So the results of the comparison are not full clear. Furthermore, the criterion of the comparison for the flapping wing in such statement is no accurate, so the results presented for the propeller can be only qualitative.

8 Concluding remarks and future work

It is well known that the constant circulation distribution is the non-optimal regime both for the propeller and the wing. So, in future, the similar task for the elliptical circulation distribution is planning to be solved. It is possible that the more clear results would be obtained.

Also it should be mentioned that we are taking into account only the power losses due to vortices. But also viscous drag must be taken into account in the analysis of efficiency.

9 Conclusions

1. The analytical model was proposed for the nonsteady effects study.
2. The nearest to the wing vortices were found to give the main contribution to the drag.
3. The method of the optimization problem solution was proposed based on the expansion of the characteristics in the Fourier series.
4. The optimal control laws were found for the different harmonics numbers. The optimal control utilization gives a little gain in comparison with sinusoidal circulation and it is smaller the motion period is longer.
5. The comparison with another variant of the thrust creation was made. It was shown that the result of comparison strongly depends on the way of ω determination.

Acknowledgment

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