Attitude Stabilization of an Indoor Quadrotor

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Abstract

This paper presents a problem of attitude stabilization and robust regulation of an indoor unmanned aerial vehicle, known as a quadrotor. The paper shows the design of continuous-time controller based on Dynamic Contraction Method. The control task is formulated as a tracking problem of Euler angles, where desired output transients are accomplished in spite of incomplete information about varying parameters of the system and external disturbances. The resulting controller has a form of a combination of a low-order linear dynamical system and a subsystem which accomplishes an algorithm of quadrotor control. Experiments results for tracking a reference signal are presented, and confirm the effectiveness of the proposed method and theoretical expectations.

1 Introduction

The always increasing performance of Micro Electro-Mechanical Systems (MEMS) inertial measurement unit (IMU), and low cost GPS have given them roles of enabling technologies for new autonomous vehicle applications. Prime examples are Unmanned Air Vehicles (UAV) for borders surveillance, forest fires monitoring, safety and natural risk management, environmental protection, management of the large infrastructures, agriculture and film production. A quadrotor, one such UAV with four fixed pitch rotors, is a highly maneuverable vehicle. It has the potential to hover and to take off, fly, and land in small areas, and has a simple control mechanism. However, a quadrotor is a complex unstable system and can be difficult to fly without modern embedded control systems. The availability of sensors and high performance small size microcontrollers have resulted in the revival of the quadrotor concept.

In practical applications, the position in space of the VTOL unmanned aerial vehicle is generally controlled by an operator through the RC transmitter using a visual feedback from an on-board camera, while the attitude is automatically stabilized via an on-board controller. The attitude controller is an important feature since it allows the vehicle to maintain a desired orientation and, hence, prevents the quadrotor from crashing when the pilot performs the desired maneuver. The attitude control problem of a VTOL-UAVs has been investigated by several researchers and a wide class of controllers has been proposed [3]–[7], [11], [13]. Thus, control of a nonlinear plant is a problem of both practical and theoretical interest.

Dynamic properties of a controlled vehicle depend on both its structure and aerodynamic qualities as well as on the control law applied. The problem of output regulation has received much attention and especially during the last decade, its nonlinear version has been intensively developed [10]. The well known approach to decoupling problem solution based on the Non-linear Inverse Dynamics (NID) method [2], [9] may be used if the parameters of the plant model and external disturbances are exactly known. Usually, incomplete information about systems in real practical tasks takes place. In this case adaptive control methods [1] or control systems with sliding mode [12] may be used for solving this control problem. A crucial feature of the sliding mode techniques is that in the sliding phase the motion of the system is insensitive to parameter variation and disturbances in the system. A way of the algorithmic solution of this problem under condition of incomplete information about varying parameters of the plant and unknown external disturbances is the application of the Localization Method (LM) [14], which allows to provide the desired transients for nonlinear time-varying systems. The generalization and development of LM is the Dynamic Contraction Method (DCM) [15]. The peculiarity of the DCM method is the application of the higher order derivatives jointly with high gain in the control law. The DCM method is insensitive to plant parameters changes and external disturbances, and works well both lineal, nonlinear, stationary and nonstationary objects.

In general, the goal of the design of a quadrotor control system is to provide decoupling of control channels in steady state, and to provide desired output transients according to the reference model.

The paper is part of a continuing effort of analytical and experimental studies on aircraft control which were considered in [8]. The main aim of this research effort is to examine the effectiveness of a designed attitude stabilization and regulation control system for quadrotor (Figure 1).

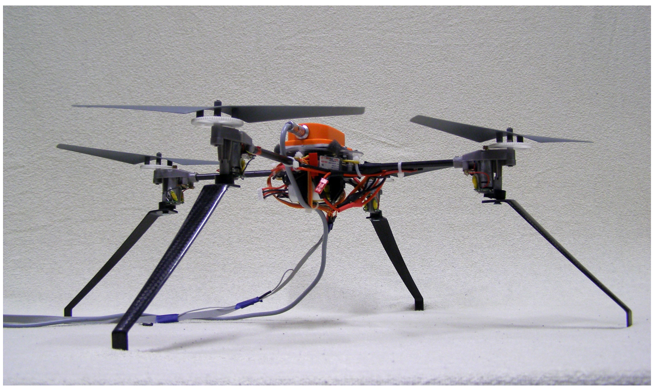

Figure 1: Quadrotor – aerial vehicle.

The paper is organized as follows. The first part includes a description of DCM method used for the control system design. Second, a mathematical description of the quadrotor model is introduced. It is desired to construct a nonlinear model of quadrotor in Simulink. The control solution along

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with the stages of regulators design are presented. It is aimed to design control system that can stabilize the attitude around hovering conditions or regulate the desired orientation, and to implement the controller using Matlab/Simulink, RTWT, PC and data acquisition card. Finally, the results of experiments in the HiL structure (Hardware in the Loop) are shown. The conclusions are briefly discussed in the last section.

2 Dynamic Contraction Method [15]

The main aim of this research effort is to design a feedback control scheme for the attitude stabilization and robust angular regulation of the quadrotor. The control system consists of two parts as shown in Figure 2. First, the main subsystem, is a MIMO controller designed using the Dynamic Contraction Method. Second subsystem accomplishes an algorithm of quadrotor control, and provides decoupling of control channels in steady state.

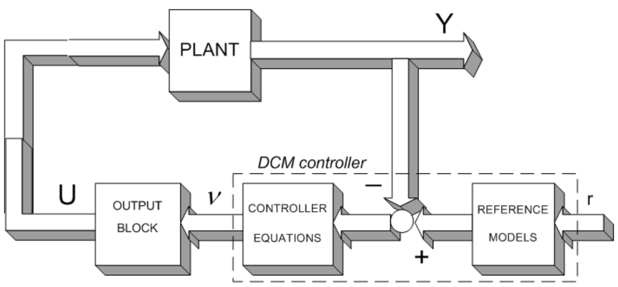

Figure 2: Block diagram of a control system.

To design a DCM controller let us consider a nonlinear time-varying MIMO system in the following form:

(1) $$\bar{x}^{(1)}(t) = \bar{h}\left(\bar{x}(t),\bar{u}(t),t\right), \quad \bar{x}(0) = \bar{x}_0$$

(2) $$\bar{y}(t) = \bar{g}\left(t,\bar{x}(t)\right),$$

where $\bar{x}(t)$ is n–dimensional state vector, $\bar{y}(t)$ is p–dimensional output vector and $\bar{u}(t)$ is l-dimensional control vector. The elements of the $\bar{f}\left(t,\bar{x}(t)\right)$, $\bar{B}\left(t,\bar{x}(t)\right)$ and $\bar{g}\left(t,\bar{x}(t)\right)$ are differentiable functions.

Each output $y_i(t)$ can be differentiated m_i times until the control input appears. Which results in the following equation:

(3) $$\bar{y}^{(m)}(t) = \bar{f}\left(t,\bar{x}(t)\right) + B\left(t,\bar{x}(t)\right)\bar{u}(t)$$

where: $\bar{y}^{(m)}(t) = \left[\, y_1^{(m_1)}, y_2^{(m_2)},\ldots, y_p^{(m_p)} \right]$,

$$\left|f_i(t,x)\right| \le f_i^{\max}, \qquad i = 1,\ 2,\ldots\ldots, p\,.$$

The value m_i is a relative order of the system (1),(2) with respect to the output $y_i(t)$ (or so called the order of a relative higher derivative). In this case the value $y_i^{(m_i)}$ depends explicitly on the input $\bar{u}(t)$.

The significant feature of the approach discussed here is that the control problem is stated as a problem of determining the root of an equation by introducing reference differential equation whose structure is in accordance with the structure of the plant model equations. So the control problem can be solved if behaviour of the $y_i^{(m_i)}$ fulfills the reference model which is given in the form of the following stable differential equations:

$$y_{1M}^{(m_1)}(t) = F_{1M}\left(\bar{y}_{1M}(t), r_1(t)\right)$$

....

(4) $$y_{iM}^{(m_i)}(t) = F_{iM}\left(\bar{y}_{iM}(t), r_i(t)\right)$$

....

$$y_{pM}^{(m_p)}(t) = F_{pM}\left(\bar{y}_{pM}(t), r_p(t)\right)$$

where: F_{iM} is called the desired dynamics of $y_i(t)$, $\bar{y}_{iM}(t) = \left[\, y_{iM}, y_{iM}^{(1)},\ldots, y_{iM}^{(m_i-1)} \right]^T$, $r_i(t)$ is the reference value and the condition $y_i = r_i$ takes place for an equilibrium point.

Denote the tracking error as follows:

(5) $$\bar{\Delta}(t) = \bar{r}(t) - \bar{y}(t)$$

The task of a control system is stated so as to provide that

(6) $$\underset{t\to\infty}{\bar{\Delta}(t)} = 0$$

Moreover, transients $\bar{y}(t)$ should have the desired behavior defined in (4) which does not depend either on the external disturbances or on the possibly varying parameters of system in equations (1), (2).

Let us denote

(7) $$\bar{\Delta}^F = \bar{F}_M\left(\bar{y}(t),\bar{r}(t)\right) - \bar{y}^{(m)}(t)$$

where: $\bar{\Delta}^F$ is the error of the desired dynamics realization, $\bar{F}_M = \left[\, F_{1M}, F_{2M},\ldots, F_{pM} \right]^T$ is a vector of desired dynamics.

As a result of (3), (4), (7) the desired behaviour of $y_i(t)$ will be provided if the following condition is fulfilled:

(8) $$\bar{\Delta}^F\left(\bar{x}(t),\bar{y}(t),\bar{r}(t),\bar{u}(t),t\right) = 0$$

So the control action $\bar{u}(t)$ which provides the control problem solution is the root of equation (8). Above expression is the insensitivity condition of the output transient performance with respect to disturbances and varying parameters of the system in (1), (2).

The solution of the control problem (8) bases on the application of the higher order output derivatives jointly with high gain in the controller. The control law in the form of a stable differential equation is constructed such that its stable equilibrium is the solution of equation (8). Such equation can be presented in the following form [10]

(9) $$\mu_i^{q_i} v_i^{(q_i)} + \sum_{j=0}^{q_i-1} \mu_i^{j} d_{i,j} v_i^{(j)} = k_i \Delta_i^F$$

$$v_i(0) = v_{i,0}$$

where: $i = 1,...,p$,

$v_i(t) = \left[v_i, v_i^{(1)}, ..., v_i^{(q_i - 1)}\right]^T$ - output of a controller,

μ_i - small positive parameter $\mu_i > 0$,

k_i - gain,

$d_{i,0}, ..., d_{i,q_i - 1}$ - diagonal matrices.

Let us assume that there is a sufficient time-scale separation, represented by a small parameter μ_i, between the fast and slow modes in the closed loop system. Methods of singularly perturbed equations can then be used to analyze the closed loop system and, as a result, slow and fast motion subsystems can be analyzed separately. Following differential equation determines the fast dynamics of controller (Figure 3):

$$D(\mu s) = \mu^q s^q + \sum_{i=0}^{q-1} \mu^i d_i s^i \tag{10}$$

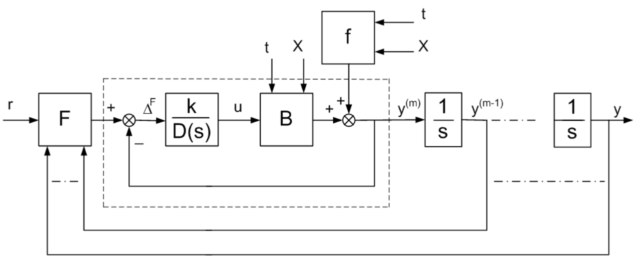

Figure 3: The main idea of a DCM theory.

Remark 1: It is assumed that the relative order of the system (1), (2), determined in (3), and reference model (4) is the same m_i .

Remark 2: Assuming that $q_i \geq m_i$ (where $i = 1,2,...,p$), then the control law (9) is proper and it can be realized without any differentiation.

Remark 3: The asymptotically stability and desired transients of $v_i(t)$ are provided by choosing $\mu_i, k_i, d_{i,0}, d_{i,1}, ..., d_{i,q_i - 1}$.

Remark 4: Assuming that $d_{i,0} = 0$ in equation (9), then the controller includes the integration and it provides that the closed-loop system is type I with respect to reference signal.

3 Quadrotor Model

The aerial vehicle consists of a rigid cross frame equipped with four rotors as shown in Figure 4.

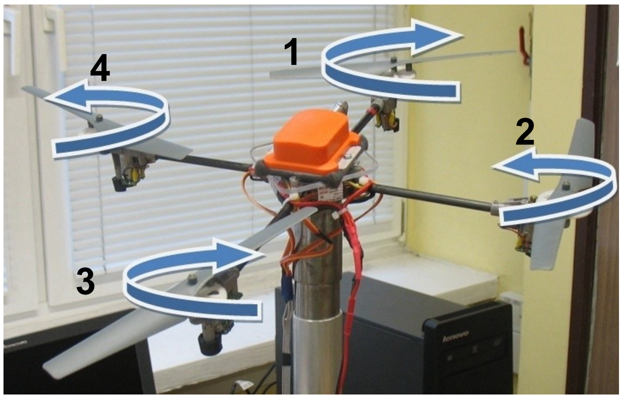

Figure 4: Quadrotor concept motion description.

The two pairs of propellers (1,3) and (2,4) turn in opposite directions. By varying the rotor speed, one can change the lift force and create motion. Thus, increasing or decreasing the four propeller's speeds together generates vertical motion. Changing the 2 and 4 propeller's speed conversely produces roll rotation coupled with lateral motion. Pitch rotation and the longitudinal motion – result from 1 and 3 propeller's speed conversely modified. Yaw rotation - as the result from the difference in the counter-torque between each pair of propellers [6].

3.1 *Mathematical Model*

The quadrotor is a six degrees of freedom system defined with twelve states. The following state and control vectors are adopted:

$$X = \left[\phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}, x, \dot{x}, y, \dot{y}, z, \dot{z}\right]^T \tag{11}$$

$$U = \left[u_1, u_2, u_3, u_4\right]^T \tag{12}$$

where:

u_i - control input of motor,

$i = 1,2,3,4$ - motor number

Six out of twelve states govern the attitude of the system (Figure 5). These include the Euler angles (ϕ, θ, ψ) and angular rates around the three orthogonal body axes. The other six states determine the position (x, y, z) and linear velocities of the center of mass of the quadrotor with respect to a fixed reference frame.

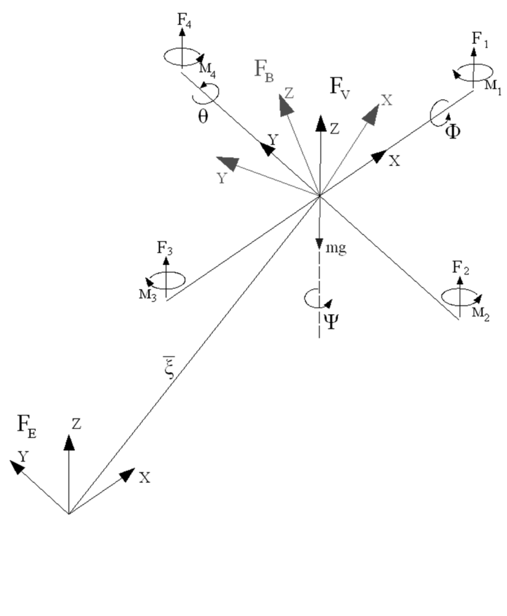

Figure 5: Quadrotor configuration.

The dynamic model is derived using Euler-Lagrange formalism [3], [4], [7] under the following assumptions:

- the frame structure is rigid,
- the structure is symmetrical,
- the CoG and the body fixed frame origin are assumed to coincide,
- the propellers are rigid,
- thrust and drag are proportional to the square of propeller's speed.

Taking this into account, the quadrotor mathematical model can be divided into two subsystems (propulsion and rigid body model) as depicted in Figure 6.

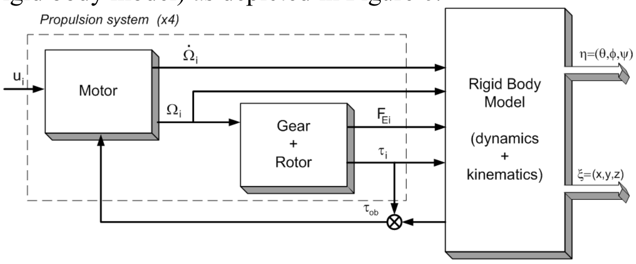

Figure 6: Block diagram of a system dynamics.

Using the Lagrangian, and the general form of the equations of motion in Lagrange method:

$$(13)\qquad L = T - V$$

$$(14)\qquad F = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q}$$

where:

L - Lagrangian,

T - kinetic energy

V - potential energy

$q = \left[x, y, z, \phi, \theta, \psi\right]^T$ - generalized coordinates

$F = \left(F_E, \tau\right)$ - generalized forces F_E and moments τ applied to the quadrotor due to the control inputs

For translational motion the Lagrange equation has a form:

$$(15)\qquad F_E = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\xi}}\right) - \frac{\partial L}{\partial \xi}$$

where:

$\xi = \left[x, y, z\right]^T$ - position coordinates,

$$F_E = \begin{bmatrix} \sin(\theta) \\ -\sin(\phi)\cos(\theta) \\ \cos(\phi)\cos(\theta) \end{bmatrix} \cdot f_g$$

$f_g = F_1 + F_2 + F_3 + F_4$

$F_i = b\Omega_i^2$

Ω_i - rotor speed

b - thrust factor

Accordingly, the Lagrange equation for rotary motion is following:

$$(16)\qquad \tau = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\eta}}\right) - \frac{\partial L}{\partial \eta}$$

where:

$\eta = \left[\phi, \theta, \psi\right]^T$ - Euler angles,

$\tau = \begin{bmatrix}\tau_\phi & \tau_\theta & \tau_\psi\end{bmatrix}^T$

$\tau_\phi = bl\left(\Omega_4^2 - \Omega_2^2\right) - J_r\dot{\theta}\left(\Omega_1 + \Omega_3 - \Omega_2 - \Omega_4\right)$

$\tau_\theta = bl\left(\Omega_3^2 - \Omega_1^2\right) + J_r\dot{\phi}\left(\Omega_1 + \Omega_3 - \Omega_2 - \Omega_4\right)$

$\tau_\psi = d\left(\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2\right)$

l - distance between propeller center and CoG

J_r - rotor inertia

d - drag factor

Above torques equations ($\tau_\phi, \tau_\theta, \tau_\psi$) consist of the action of the thrust forces difference of each pair, and from the gyroscopic effect.

Finally the quadrotor dynamic model with x,y,z, motions as a consequence of a pitch, roll and yaw rotations is as follows:

$$(17)\qquad \ddot{\theta} = \frac{1}{I_{xx}}\left(-\dot{\phi}^2\left(I_{xx} - I_{zz}\right)s(\theta)c(\theta) - \dot{\phi}\dot{\psi}I_{zz}c(\theta) + \tau_\theta\right)$$

$$(18)\qquad \ddot{\phi} = \frac{1}{I_{yy}\left(1 + s^2(\theta)\right)}\left(-\ddot{\psi}I_{zz}s(\theta) - \dot{\theta}\dot{\phi}c(\theta)s(\theta)\cdot\right.$$
$$\left.\cdot\left(2I_{zz} - 2I_{yy}\right) - \dot{\theta}\dot{\psi}I_{zz}c(\theta) + \tau_\phi\right)$$

$$(19)\qquad \ddot{\psi} = \frac{1}{I_{zz}}\left(-\ddot{\phi}I_{zz}s(\theta) + \tau_\psi\right)$$

$$(20)\qquad \ddot{x} = \frac{f_g}{m}s(\theta)$$

$$(21)\qquad \ddot{y} = -\frac{f_g}{m}c(\theta)s(\phi)$$

$$(22)\qquad \ddot{z} = \frac{f_g}{m}c(\theta)c(\phi) - g$$

where: s and c are abbreviations of '*sin*' and '*cos*',

I_{xx}, I_{yy}, I_{zz} - inertia moments.

The complete rigid body model can be presented in the form of block diagram (Figure 7). The internal block contains moments and forces generated by quadrotor, linear and angular velocity, and the attitude and position which can be calculated on account of the inputs from the block of propulsion system.

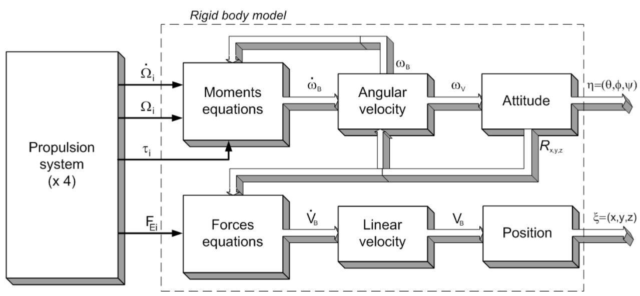

Figure 7: Block diagram of the rigid body model.

3.2 *Propulsion system*

The model of brushed DC-motor is divided into an electrical and mechanical part:

$$L\frac{dI}{dt} = u - RI - k_e\omega_m \tag{23}$$

$$J_m\frac{d\omega_m}{dt} = \tau_m - \tau_l \tag{24}$$

where:

- L - motor inductance
- I - motor current
- u - motor input
- R - motor resistance
- k_e - motor electrical constant
- ω_m - motor angular speed
- J_m - motor inertia
- $\tau_m = k_m I$ - motor torque
- k_m - torque constant
- τ_l - motor load

Because of the very low inductance of used small engines, the DC-motor dynamics can be approximated by the first order differential equation in the following form:

$$J_m\frac{d\omega_m}{dt} = \frac{k_m}{R}u - \frac{k_m k_e}{R}\omega_m - \tau_l \tag{25}$$

Block diagram of a motor dynamics is to be assembled from the above derivation and the result is in Figure 8.

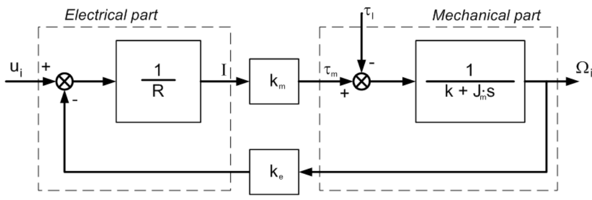

Figure 8. Block diagram of a DC-motor dynamics.

4 Attitude Control System Design

The entire closed loop system is presented in Figure 2. The quadrotor model described by equations (11)–(25), will be used to design the attitude control system that achieves the angular stabilization and regulation by tracking a reference signal. The control task is stated as a tracking problem for the following variables:

$$\begin{aligned} &\lim_{t\to\infty}\left[\phi_0(t) - \phi(t)\right] = 0 \\ &\lim_{t\to\infty}\left[\theta_0(t) - \theta(t)\right] = 0 \\ &\lim_{t\to\infty}\left[\psi_0(t) - \psi(t)\right] = 0 \end{aligned} \tag{26}$$

where $\phi_0(t), \theta_0(t), \psi_0(t)$ are the desired values of the considered variables.

In addition, we require that transient processes have desired dynamic properties, are mutually independent and are independent of quadrotor parameters and disturbances.

Feedback data for the regulator are three variables: ϕ, θ, ψ. Control signals are motors inputs: u_1, u_2, u_3, u_4. In this case:

$$\dim(\bar{y}) \neq \dim(\bar{u}) \tag{27}$$

Therefore the control system is divided into two subsystems: first – MIMO DCM controller, second – output block.

4.1 DCM controller

The inverse dynamics of (1), (2) are constructed by differentiating the individual elements of $\bar{y}$ sufficient number of times until a term containing $\bar{u}$ appears in (3). From equations of quadrotor motion (17)–(25), and following (3), the below relationship becomes:

$$\begin{bmatrix} \phi^{(2)} \\ \theta^{(2)} \\ \psi^{(2)} \end{bmatrix} = \begin{bmatrix} f_\phi \\ f_\theta \\ f_\psi \end{bmatrix} + B\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \tag{28}$$

Let us assume that the desired dynamics are determined by a set of mutually independent differential equations:

$$\tau_\phi^2\phi^{(2)} = -2\tau_\phi\alpha_\phi\phi^{(1)} - \phi + \phi_0 \tag{29}$$

$$\tau_\theta^2\theta^{(2)} = -2\tau_\theta\alpha_\theta\theta^{(1)} - \theta + \theta_0 \tag{30}$$

$$\tau_\psi^2\psi^{(2)} = -2\tau_\psi\alpha_\psi\psi^{(1)} - \psi + \psi_0 \tag{31}$$

Parameters τ_i and α_i $(i = \phi, \theta, \psi)$ have very well known physical meaning and their particular values have to be specified by the designer.

The dynamic part of the control law from (9) has the following form:

$$\begin{aligned} &\mu_\phi^2 v_\phi^{(2)} + 2d_{\phi,1}\mu_\phi v_\phi^{(1)} + d_{\phi,0}v_\phi = \\ &\qquad k_\phi\left(-\tau_\phi^2\phi^{(2)} - 2\alpha_\phi\tau_\phi\phi^{(1)} - \phi + \phi_0\right) \end{aligned} \tag{32}$$

$$\begin{aligned} &\mu_\theta^2 v_\theta^{(2)} + 2d_{\theta,1}\mu_\theta v_\theta^{(1)} + d_{\theta,0}v_\theta = \\ &\qquad k_\theta\left(-\tau_\theta^2\theta^{(2)} - 2\alpha_\theta\tau_\theta\theta^{(1)} - \theta + \theta_0\right) \end{aligned} \tag{33}$$

$$\begin{aligned} &\mu_\psi^2 v_\psi^{(2)} + 2d_{\psi,1}\mu_\psi v_\psi^{(1)} + d_{\psi,0}v_\psi = \\ &\qquad k_\psi\left(-\tau_\psi^2\psi^{(2)} - 2\alpha_\psi\tau_\psi\psi^{(1)} - \psi + \psi_0\right) \end{aligned} \tag{34}$$

4.2 Output block

The main goal of the second subsystem is to accomplish an algorithm of quadrotor control, and provides decoupling of control channels in steady state. Thus the control inputs from DCM controller, about each axis v_ϕ, v_θ, v_ψ, are combined to generate the control inputs u_1 through u_4, for motors 1 through 4:

$$u_1 = u_{th} + v_\theta + v_\psi \tag{35}$$

$$u_2 = u_{th} + v_\phi - v_\psi \tag{36}$$

$$u_3 = u_{th} - v_\theta + v_\psi \tag{37}$$

$$u_4 = u_{th} - v_\phi - v_\psi \tag{38}$$

where:

- v_ϕ, v_θ, v_ψ - control inputs about axis x, y, z, respectively,
- u_{th} - collective control command for each motor.

5 STRUCTURE AND TEST BENCH

The development of a attitude control system for quadrotor requires the development of an adequate test-bench for needs of the controller tuning, at least for the initial experiments. It can help to lock three degrees of freedom in order to reduce tuning controller complexity and avoid system damage. For the preliminary control experiments we use the test-bench (Figure 9), which consists of: (1) quadrotor airframe with propulsion system, (2) AHRS, (3) PC with I/O card, (4) power supply, (5) RC transmitter, (6) stand. The frame composed of carbon tubes attached to a plastic hub, and at the other ends with propulsion systems. To the airframe added AHRS sensor and electronic circuitry. The miniature MTi Xsens AHRS estimates with a Kalman filter the 3D orientation data and gives the calibrated data of acceleration and angular velocity.

It weights 50g and communicates at 115kbps by the external converter RS-232/USB. For needs of the controller tuning, it was decided to use a stationary ball joint base, as shown in Fig.9. This stand gives the quadrotor unrestricted yaw movement and around $\pm 50^o$ pitch and roll angles. The height is around 1.7m, and it eliminates the influence of disturbances in the form of air stream reflection. The quadrotor has four propulsion systems, each one is composed of a brushed DC-motor driven by PWM signal, gear box with a speed reduction ratio of 9:1, and two-bladed propeller. Experimental testing has been performed, with a sampling frequency of 1 kHz, using a stationary PC with I/O card Inteco RT-DAC4 as a Data Acquisition and Control Device. The Matlab and Simulink software in combination with Real-Time Workshop and RT-CON allows an easy implementation of the control system in block diagram format via Simulink, with real-time tuning the controller parameters. This structure of experimental setup was used for a fast prototyping of designed DCM controller, as well as the attitude control system concept, in the hardware in the loop system.

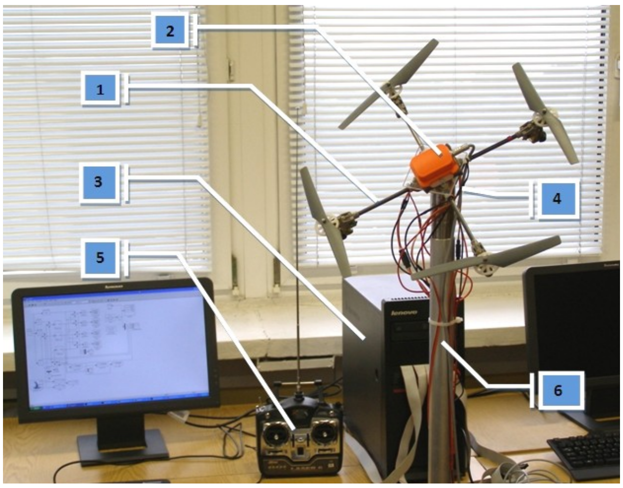

Figure 9: Quadrotor experimental setup.

In practice, the motion of the quadrotor in three-dimensional space is achieved by the operator (onboard or through a RC transmitter) by specifying the desired total thrust, roll, pitch and yaw, while the attitude is automatically controlled according to the algorithms proposed in previous section.

The entire closed loop scheme with attitude control system is shown in Figure 10.

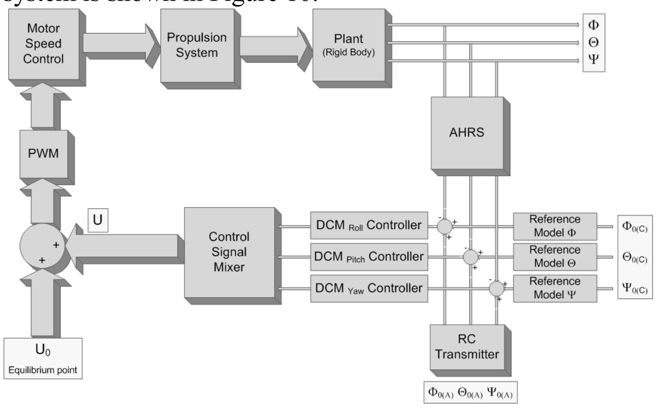

Figure 10: Control implementation.

6 RESULTS OF CONTROL EXPERIMENTS

In this section, we present the results of experiment which was conducted on the quadrotor, to evaluate the performance of a designed attitude control system.

At first, in order to predict the performance of quadrotor, a simulation system was developed under Matlab/Simulink platform. This helpful tool was used to determine the reference models for each control channel. The significant task ,during the angular stabilization, fulfill the roll and pitch channels, therefore the dynamics of reference models for both angles were assumed faster than yaw channel.

Next step includes the angular stabilization, regulation, and investigation in area of robustness performed on the test-bench. The presented maneuver consisted in transition with predefined dynamics from one steady-state angular position to another. Hereby, the control system accomplished a tracking task of reference signal. The experiment was chosen to expose a robustness of the controller under transient and steady-state conditions. During the experiment, the entire control system was subjected to external disturbances in the form of a wind gust. Practically this perturbation was realized mechanically by pushing the quadrotor frame in different directions ($t \in (50,65)$ [s] and $t \in (95,105)$ [s]).

After several simulations and tests performed on the experimental setup, it was time to test an autonomous flight. First flights were successful without additional tuning the controller parameters. Only the influence of some perturbations, introduced by the sensor and control cables, were observable.

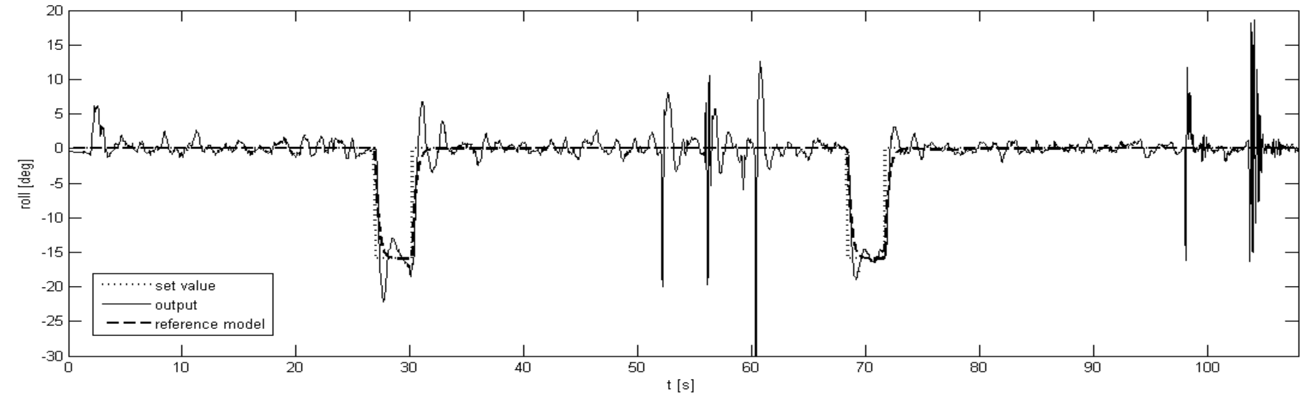

Figure 11: Time history of roll angle ϕ *[deg]*

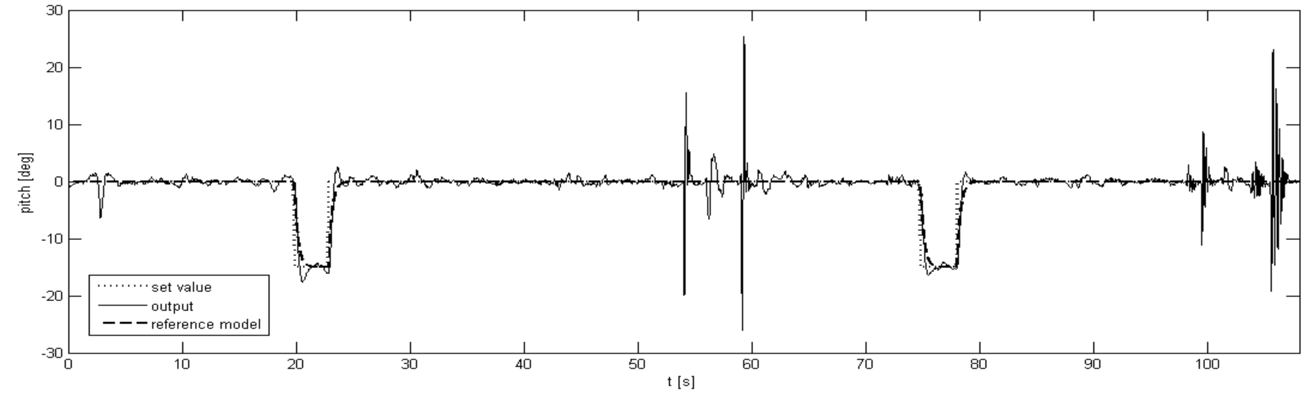

Figure 12: Time history of pitch angle θ *[deg]*

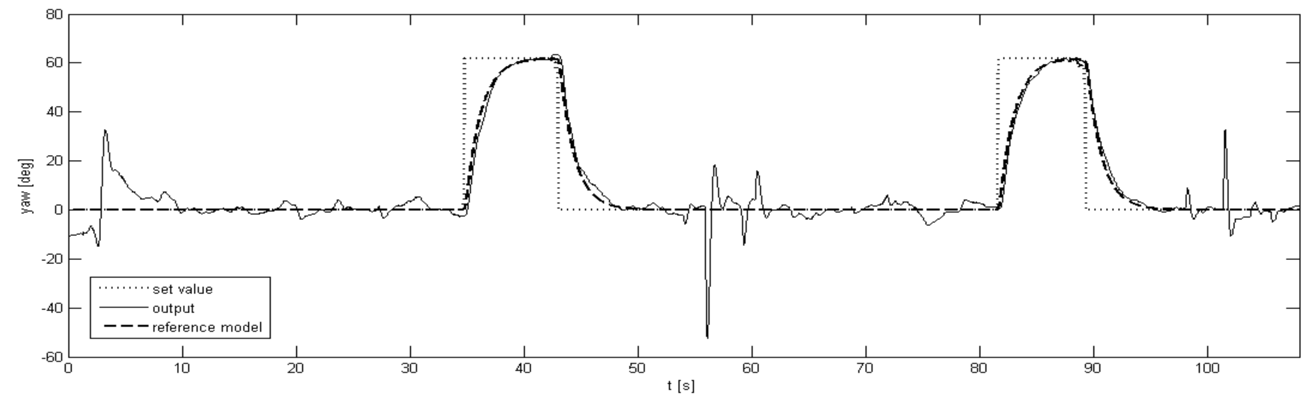

Figure 13: Time history of yaw angle ψ *[deg]*

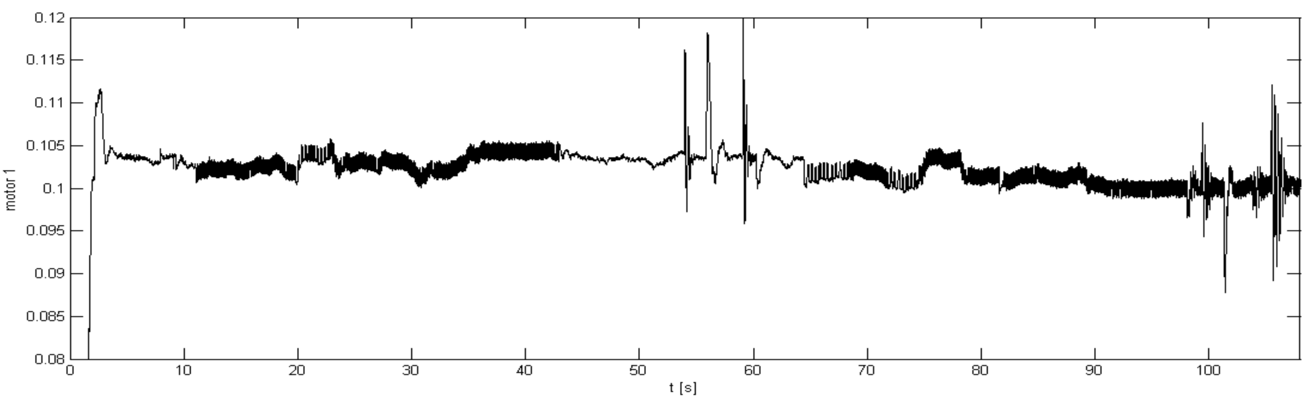

Figure 14: Time history of control signal u_1

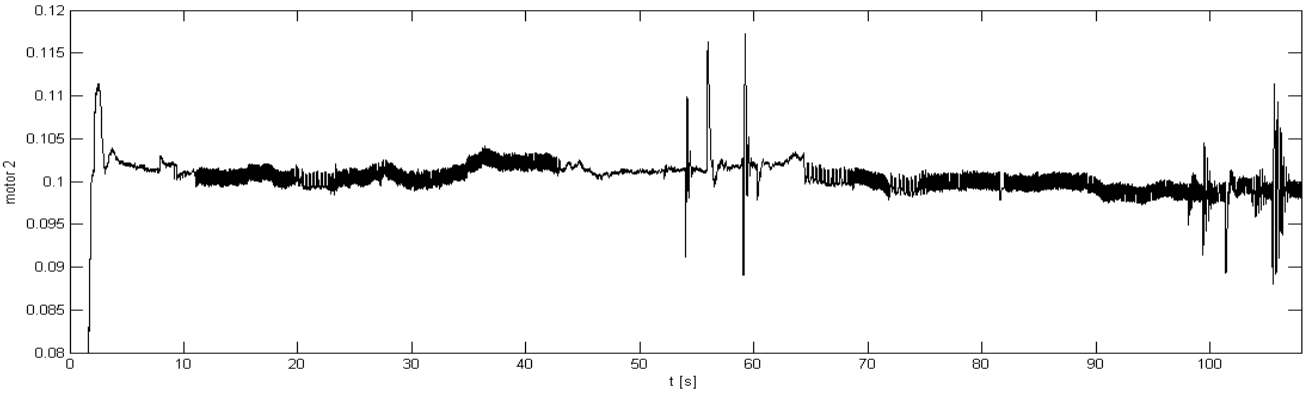

Figure 15: Time history of control signal u_2

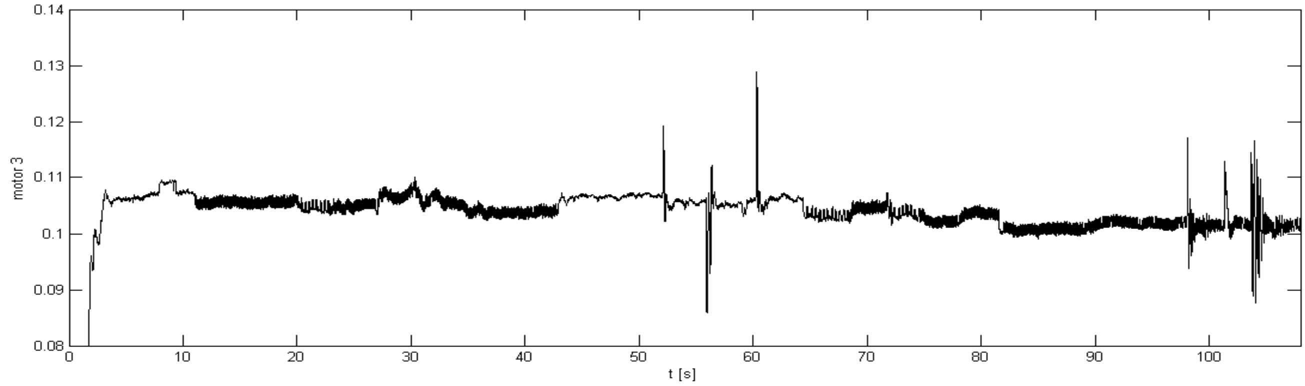

Figure 16: Time history of control signal u_3

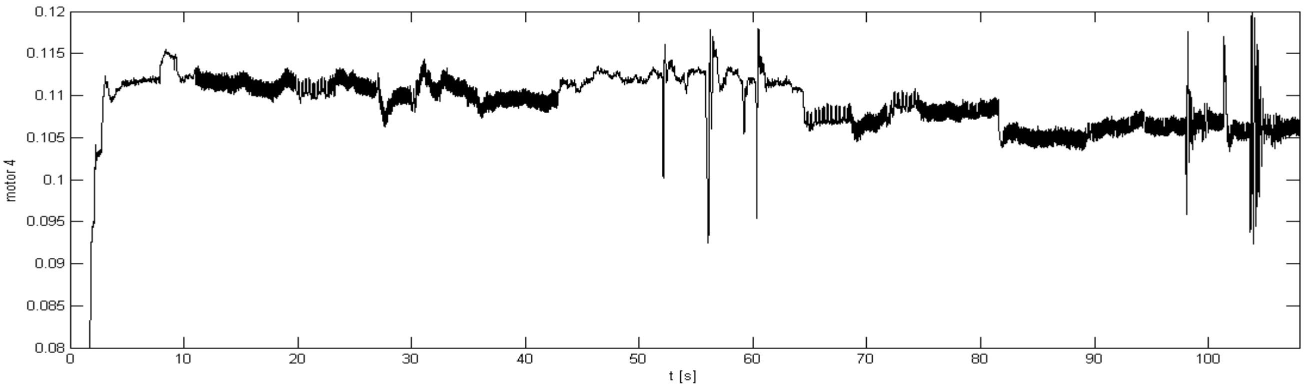

Figure 17: Time history of control signal u_4

7 CONCLUSION

Autonomous quadrotor can make possible many potential applications for unmanned aerial vehicles. In this paper, the problem of attitude stabilization and regulation of an indoor unmanned aerial vehicle, known as a quadrotor, was considered. The paper includes vehicle dynamic modeling, investigation of DCM theory, MIMO controller design, experiments. The applied DCM theory allows to create the expected outputs for multi-input multi-output nonlinear time-varying physical object, like an quadrotor, and provides independent desired dynamics in control channels. The peculiarity of the propose approach is the application of the higher order derivatives jointly with high gain in the control law. This approach and structure of the control system is the implementation of the model reference control with the reference model transfer function which is equal to the inverse of the controller "dynamics". The DCM technique was explored from theoretical development to final experiments. It becomes that the proposed structure and method is insensitive to external disturbances and also plant parameter changes, and hereby possess a robustness aspects. Simulations and test-bench experiments prove the ability, of the designed DCM regulator, to control the orientation angles in the presence of perturbations. The successful first autonomous flight validates the previous all stages of attitude control system design.

ACKNOWLEDGMENT

This work has been granted by the Polish Ministry of Science and Higher Education from funds for years 2008-2011.

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